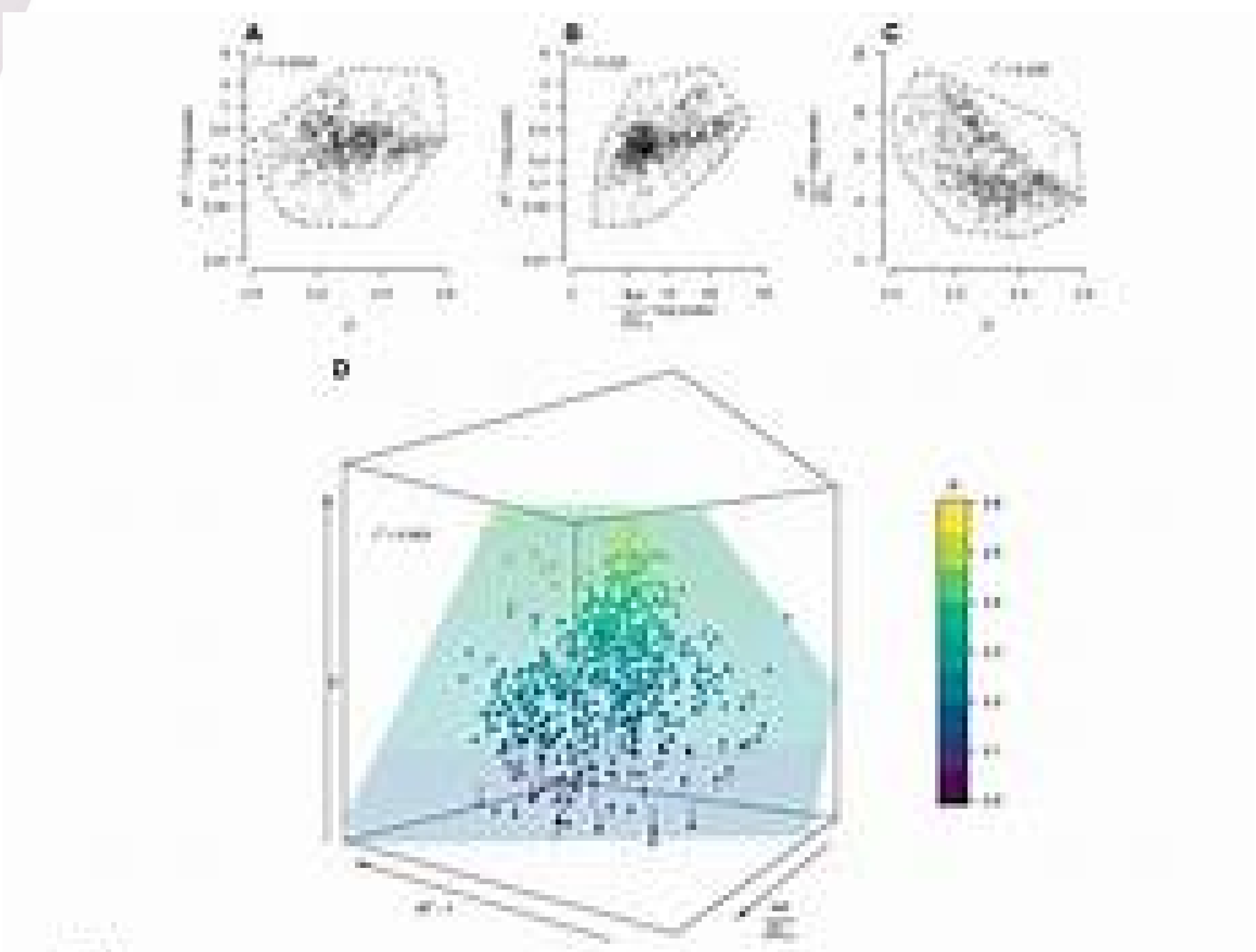
MEASURE THEORY & INTEGRATION



Mrs. M. Meenakshi

MEASURE THEORY & INTEGRATION

MULTIPLE CHOICE QUESTIONS WITH ANSWERS

UNIT-I

1. Which mathematician introduced the concept of Lebesgue measure?

a. Henri Lebesgue b. Carl Gauss

c. Leonhard Euler d. Bernhard Riemann

Hint: This mathematician is known for his work on integration.

2. What does Lebesgue measure aim to generalize?

a. Riemann integration

b. Fourier series

c. Differential equations

d. Taylor series

Hint: It's a method of measuring sets.

3. The concept of outer measure extends the notion of:

a. Length b. Area

c. Volume d. All of the above

Hint: It is a broad concept encompassing various geometrical measurements.

4. The outer measure of an empty set is:

a. Zero b. Infinity

c. Undefined d. One

Hint: Consider the fundamental properties of sets.

5. Which property does the outer measure possess?

a. Subadditivity b. Additivity

c. Multiplicativity d. Division

Hint: It's a property that involves the combination of measures.

6. The measure of a countable union of disjoint sets is equal to:

a. The sum of their individual measures

b. The maximum of their measures

c. The minimum of their measures

d. The average of their measures

Hint: Think about how the measures combine in unions.

7. The Lebesgue measure of an interval [a, b] on the real line is:

a. b – a   b. b + a

c. (b - a)^2   d. 2b - 2a

Hint: Consider how to measure the length of an interval.

8. Which set has Lebesgue measure zero?

a. A single point
b. An open interval
c. A closed interval
d. A half-open interval

Hint: Focus on the concepts of size and measure.

9. The Cantor set is an example of a set with:

a. Finite Lebesgue measure
b. Countably infinite Lebesgue measure
c. Uncountably infinite Lebesgue measure
d. Lebesgue measure equal to 1

Hint: Consider the construction and properties of the Cantor set.

10. Which of the following is a property of Lebesgue measurable sets?

a. Countable additivity
b. Uncountable additivity
c. Finite additivity
d. Infinite additivity

Hint: Focus on the way measurable sets combine.

11. Which set is always Lebesgue measurable?

a. Any open set
b. Any closed set
c. Any bounded set
d. Any countable set

Hint: Think about the properties of sets that make them easily measurable.

12. The Lebesgue measure is translation-invariant. What does this mean?

a. The measure of a translated set is the same as the original set.
b. The measure of a set depends on its translation.
c. Translations do not affect the measure of sets.
d. The measure of a translated set is double the original set.

Hint: Consider the impact of translation on measurement.

13. The concept of Lebesgue measure is essential in:

a. Quantum mechanics
b. General relativity
c. Measure theory
d. Number theory

Hint: Focus on the field where measures and integration play a crucial role.

14. Which function is NOT integrable with respect to the Lebesgue measure?

a. Indicator function of a finite set
b. Indicator function of an open set
c. Indicator function of a closed set
d. Indicator function of an uncountable set

Hint: Consider the properties of integrable functions in Lebesgue measure.

15. The Lebesgue measure of a countable union of disjoint intervals in R is equal to:

a. The sum of the lengths of the intervals
b. The maximum length of the intervals
c. The minimum length of the intervals
d. The average length of the intervals

Hint: Consider how intervals combine in unions.

16. Which of the following sets has Lebesgue measure 1 in the unit interval [0, 1]?

a. The set of rational numbers in [0, 1]
b. The set of irrational numbers in [0, 1]
c. The set of integers in [0, 1]
d. The set of real numbers in [0, 1]

Hint: Think about the nature of rational and irrational numbers.

17. The outer measure of a subset of a set is always:

a. Greater than or equal to the outer measure of the set
b. Less than or equal to the outer measure of the set
c. Equal to the outer measure of the set
d. Unrelated to the outer measure of the set

Hint: Consider the relationship between sets and their subsets.

18. Which operation always preserves Lebesgue measurability?

a. Union
b. Intersection
c. Complement
d. None of the above

Hint: Think about how the properties of sets change with operations.

19. Which of the following sets is not Lebesgue measurable?

a. The set of all algebraic numbers
b. The set of all transcendental numbers
c. The set of all integers
d. The set of all real numbers

Hint: Consider the properties of algebraic and transcendental numbers.

20. The concept of outer measure is a precursor to the development of:

a. Fourier series
b. Lebesgue integration
c. Complex analysis
d. Differential equations

Hint: Think about the mathematical concepts that build upon the idea of outer measure.

21. What defines a set as Lebesgue measurable?

a. It has a well-defined area or volume
b. It can be approximated by rectangles
c. It satisfies the Carathéodory criterion
d. It can be split into countably many parts

Hint: Think about the property required for sets to have a measure.

22. Which of the following sets is NOT necessarily Lebesgue measurable?

a. Closed intervals
b. Open intervals
c. Bounded sets
d. Any arbitrary set

Hint: Consider the properties that guarantee measurability.

23. The Lebesgue measurable sets form a:

a. Vector space
b. Field
c. σ-algebra
d. Topological space

Hint: Think about the properties of sets in Lebesgue measure theory.

24. A set is Lebesgue measurable if and only if it is:

a. Countable
b. A union of open intervals
c. The complement of an open set
d. The union of a null set and a Borel set

Hint: Consider the defining property for Lebesgue measurability.

25. The Lebesgue outer measure of a set is always:

a. Greater than or equal to zero
b. Less than or equal to zero
c. Greater than or equal to one
d. Less than or equal to one

Hint: Reflect on the nature of outer measures.

26. Which of the following statements is true regarding Lebesgue measure and translation invariance?

a. The measure of a translated set differs from the original set.
b. The measure of a translated set is equal to the original set.
c. Translations affect the measure of sets unpredictably.
d. Translation invariance doesn't apply to Lebesgue measure.

Hint: Consider the effect of translation on measures.

27. The Cantor set has Lebesgue measure:

a. Zero
b. One
c. Infinity
d. Undefined

Hint: Recall the properties of the Cantor set and its measure.

28. Which of these is always a Lebesgue measurable set?

a. Countable union of measurable sets
b. Countable intersection of measurable sets
c. Countable disjoint union of measurable sets
d. Countable complement of a measurable set

Hint: Consider the properties preserved in different set operations.

29. The Lebesgue measure of an open interval (a, b) in {R} is:

a. b - a
b. b + a
c. (b - a)^2
d. 2b - 2a

Hint: Think about how to measure the length of an interval.

30. A set is Lebesgue measurable if its outer measure is equal to:

a. Zero
b. Infinity
c. Any finite positive number
d. A negative number

Hint: Consider the relationship between outer measure and measurability.

31. The Lebesgue measure of a countable set is always:

a. Zero
b. Infinite
c. Undefined
d. Equal to the cardinality of the set

Hint: Consider the property of countable sets in Lebesgue measure.

Answer: A) Zero

32. Which of the following statements is NOT true about Lebesgue measurable sets?

a. They form a σ-algebra.
b. They are closed under countable unions.
c. They are closed under countable intersections.
d. They are closed under complements.

Hint: Reflect on the properties of Lebesgue measurable sets.

33. The union of two disjoint Lebesgue measurable sets is:

a. Always Lebesgue measurable
b. Not necessarily Lebesgue measurable

c. Lebesgue measurable if their intersection is empty
d. Always of infinite measure

Hint: Consider the property preserved in unions of measurable sets.

34. The Lebesgue measure is defined on which class of sets?

a. All subsets of {R}
b. All Borel sets in {R}
c. All countable sets in {R}
d. All open sets in {R}

Hint: Consider the specific class of sets on which the measure is defined.

35. Which of the following is true regarding Lebesgue measure and countable additivity?

a. It applies to all sets.
b. It applies only to Borel sets.
c. It applies only to open sets.
d. It applies only to measurable sets.

Hint: Reflect on the property of countable additivity.

36. The Lebesgue measure of a countable union of disjoint intervals in \(\mathbb{R}\) is equal to:

a. The sum of the lengths of the intervals
b. The maximum length of the intervals
c. The minimum length of the intervals
d. The average length of the intervals

Hint: Consider how intervals combine in unions.

37. Which of the following sets is not necessarily Lebesgue measurable?

a. The set of rational numbers
b. The set of irrational numbers
c. The set of algebraic numbers
d. The set of transcendental numbers

Hint: Consider the properties of different types of numbers.

38. The Lebesgue measure is a(n):

a. Outer measure
b. Inner measure
c. Discrete measure
d. Continuous measure

Hint: Reflect on the concept of measure.

39. Which property does the Lebesgue measure possess?

a. Subadditivity
b. Additivity
c. Multiplicativity
d. Division

Hint: It's a property that involves the combination of measures.

40. The concept of Lebesgue measure is primarily applied in:

a. Number theory
b. Quantum mechanics
c. Measure theory
d. General relativity

Hint: Think about the field where measures and integration are crucial.

41. Littlewood's First Principle states that any function that is a limit of a sequence of simple functions is:

a. Bounded
b. Continuous
c. Measurable
d. Integrable

Hint: Think about the property required for functions to satisfy this principle.

42. A function is said to be measurable if:

a. It is continuous everywhere
b. Its graph is continuous
c. Its preimage of every measurable set is measurable
d. It is differentiable

Hint: Consider the property that determines the measurability of functions.

43. Littlewood's Second Principle deals with the measurability of:

a. Functions on compact sets
b. Functions on closed intervals
c. Real-valued functions
d. Functions on measurable sets

Hint: Focus on the property related to measurable functions.

44. Which of the following is true according to Littlewood's Third Principle?

a. A pointwise limit of measurable functions is measurable
b. A pointwise limit of continuous functions is measurable
c. A pointwise limit of bounded functions is measurable

d. A pointwise limit of integrable functions is measurable

Hint: Consider the property needed for a limit of functions to be measurable.

45. Littlewood's Three Principles pertain to the concept of:

a. Continuous functions
b. Measurable functions
c. Bounded functions
d. Differentiable functions

Hint: Focus on the specific property addressed by these principles.

46. A function is said to be measurable if its preimage of every open set is:

a. Bounded
b. Open
c. Closed
d. Measurable

Hint: Consider the property that defines the measurability of functions.

47. Which type of convergence preserves measurability according to Littlewood's Principles?

   a. Pointwise convergence
   b. Uniform convergence
   c. Absolute convergence
   d. Conditional convergence

Hint: Reflect on the kind of convergence required to maintain measurability.

48. Littlewood's Second Principle emphasizes the importance of:

   a. Functions on compact sets
   b. Functions on open sets
   c. Functions on measurable sets
   d. Functions on closed sets

Hint: Focus on the sets with respect to which the principle is stated.

49. According to Littlewood's First Principle, a sequence of simple functions converging to a measurable function implies the measurable function is:

   a. Continuous
   b. Integrable
   c. Measurable
   d. Bounded

Hint: Consider the property required for the limit function in this context.

50. Littlewood's Third Principle focuses on the behavior of:

a. Continuous functions
b. Integrable functions
c. Bounded functions
d. Pointwise limits of functions

Hint: Consider the property needed for pointwise limits of functions.

51. Littlewood's Three Principles are mainly concerned with the properties of functions in the context of:

a. Topology
b. Measure theory
c. Real analysis
d. Complex analysis

Hint: Consider the area of mathematics these principles apply to.

52. A function f is said to be measurable if the preimage of every closed set is:

a. Measurable
b. Open
c. Bounded

d. Continuous

Hint: Reflect on the property related to the preimage of sets.

53. Littlewood's First Principle establishes the relationship between:

a. Simple functions and integrable functions
b. Continuous functions and measurable functions
c. Bounded functions and unbounded functions
d. Measurable functions and non-measurable functions

Hint: Consider the relationship discussed in the first principle.

54. The set of all measurable functions forms a:

a. Vector space
b. Field
c. σ-algebra
d. Topological space

Hint: Consider the properties of measurable functions as a collection.

55. Littlewood's Second Principle emphasizes the importance of functions on:

a. Open sets

b. Closed sets
c. Compact sets
d. Bounded sets

Hint: Focus on the type of sets involved in the second principle.

56. Measurable functions play a significant role in the context of:

a. Algebra
b. Topology
c. Measure theory
d. Number theory

Hint: Consider the mathematical field where measurable functions are crucial.

57. Littlewood's Third Principle deals with the property of:

a. Boundedness of functions
b. Integrability of functions
c. Pointwise limits of functions
d. Continuity of functions

Hint: Focus on the property addressed by the third principle.

58. Littlewood's Principles address the behavior of functions concerning:

a. Integration
b. Differentiation
c. Measure
d. Continuity

Hint: Reflect on the specific property discussed in these principles.

59. According to Littlewood's First Principle, the limit of a sequence of simple functions is always:

a. Measurable
b. Integrable
c. Continuous
d. Bounded

Hint: Consider the property required for the limit function in this context.

60. Littlewood's Third Principle is related to the convergence of:

a. Bounded functions
b. Continuous functions
c. Integrable functions
d. Pointwise limits of functions

Hint: Focus on the type of convergence discussed in the third principle.

61. A function is said to be measurable if its preimage of every Borel set is:

a. Continuous
b. Integrable
c. Measurable
d. Differentiable

Hint: Consider the property related to the preimage of sets.

62. Littlewood's First Principle deals with the relationship between:

a. Integrable functions and measurable functions
b. Continuous functions and integrable functions
c. Simple functions and measurable functions
d. Measurable functions and non-measurable functions

Hint: Reflect on the relationship discussed in the first principle.

63. Littlewood's Second Principle addresses the behavior of functions on:

a. Open sets
b. Closed sets

c. Compact sets
d. Infinite sets

Hint: Focus on the kind of sets involved in the second principle.

64. According to Littlewood's Third Principle, a pointwise limit of measurable functions is always:

a. Continuous
b. Integrable
c. Measurable
d. Bounded

Hint: Consider the property required for the limit function in this context.

65. Littlewood's Three Principles primarily concern the properties of functions with respect to:

a. Topological spaces
b. Measure spaces
c. Vector spaces
d. Metric spaces

Hint: Reflect on the spaces in which these principles apply.

66. Littlewood's First Principle emphasizes the importance of:

a. Integrable functions
b. Measurable functions
c. Continuous functions
d. Bounded functions

Hint: Focus on the property discussed in the first principle.

67. Measurable functions are important in the context of:

a. Real analysis
b. Complex analysis
c. Number theory
d. Topology

Hint: Consider the area of mathematics where measurable functions are crucial.

68. According to Littlewood's Second Principle, functions on measurable sets are important concerning their behavior on:

a. Open sets
b. Closed sets
c. Bounded sets
d. Compact sets

Hint: Focus on the type of sets involved in the second principle.

69. Littlewood's Third Principle deals with the behavior of:

a. Continuous functions
b. Integrable functions
c. Bounded functions
d. Pointwise limits of functions

Hint: Consider the property addressed by the third principle.

70. Littlewood's Principles are primarily concerned with the properties of functions in the context of:

a. Integration
b. Measure theory
c. Topology
d. Differentiation

Hint: Reflect on the specific mathematical field where these principles apply.

71. Littlewood's Three Principles refer to:

a. Principles of integration
b. Principles of measure theory
c. Principles of approximation
d. Principles of convergence

Hint: Littlewood's Three Principles focus on specific aspects related to measure theory.

72. Which of the following is a characteristic of a measurable set according to the Lebesgue measure?

a. Countable additivity
b. Inner regularity
c. Approximation by open sets
d. Finite additivity

Hint: Think about the defining properties of measurable sets according to the Lebesgue measure.

73. In measure theory, a set is considered measurable if it satisfies:

a. The Borel-Cantelli lemma
b. The Heine-Borel theorem
c. Carathéodory's criterion
d. Fatou's lemma

Hint: Consider the criteria that define a measurable set according to measure theory.

74. Which of the following statements is true about the Lebesgue measure?

a. It is finitely additive
b. It is countably sub additive
c. It is defined on all subsets of the real numbers

d. It is only defined for open sets

Hint: Recall the specific properties and scope of the Lebesgue measure.

75. Littlewood's First Principle involves:

a. Uniform continuity
b. Uniform convergence
c. Pointwise convergence
d. Pointwise continuity

Hint: Think about the type of convergence that Littlewood's First Principle addresses.

76. Measurable sets in measure theory are closed under:

a. Finite intersections
b. Arbitrary unions
c. Finite unions
d. Complements

Hint: Consider the closure properties of measurable sets in measure theory.

77. The Lebesgue measure of the empty set is:

a. Zero
b. Undefined
c. Infinity
d. Not measurable

Hint: Recall the properties of the Lebesgue measure, especially regarding the empty set.

78. Littlewood's Third Principle focuses on:

a. Convergence almost everywhere
b. Convergence in measure
c. Uniform convergence
d. Pointwise convergence

Hint: Consider the type of convergence addressed by Littlewood's Third Principle.

79. The concept of measurability in measure theory is related to the:

a. Density of sets
b. Approximation of sets
c. Continuity of functions
d. Integrability of functions

Hint: Consider the defining characteristics of measurable sets in measure theory.

80. Littlewood's Second Principle deals with the properties of:

a. Dense sets
b. Negligible sets
c. Open sets
d. Convergent sets

Hint: Think about sets that are related to the concepts of Littlewood's Principles.

81. Which property characterizes a set as negligible in measure theory?

a. It has zero Lebesgue measure
b. It is finite
c. It is closed
d. It has a Lebesgue measure of infinity

Hint: Consider the defining property of negligible sets in measure theory.

82. Littlewood's Principles are concerned with the behavior of functions and sets concerning their:

a. Differentiability
b. Integrability
c. Approximation
d. Continuity

Hint: Focus on the aspects of functions and sets addressed by Littlewood's Principles.

83. A set that is both measurable and negligible according to the Lebesgue measure is:

a. Empty set

b. Singleton set
c. Closed set
d. Dense set

Hint: Consider the properties of measurable and negligible sets together.

84. The Lebesgue measure is defined on which sets?

a. All subsets of a measure space
b. Only open sets
c. Borel sets
d. Closed sets

Hint: Consider the specific classes of sets on which the Lebesgue measure is defined.

85. Which property characterizes Littlewood's Third Principle regarding convergence?

a. Uniform convergence
b. Convergence almost everywhere
c. Pointwise convergence
d. Convergence in measure

Hint: Focus on the specific type of convergence addressed by Littlewood's Third Principle.

ANSWERS

S.NO	OPTIONS
1	A
2	A
3	D
4	A
5	A
6	A
7	A
8	A
9	A
10	A
11	A
12	C
13	C
14	D
15	A
16	B
17	B
18	A
19	C
20	B
21	C
22	D
23	C
24	D
25	A
26	B

27	A
28	A
29	A
30	A
31	A
32	C
33	A
34	B
35	D
36	A
37	A
38	A
39	B
40	C
41	C
42	C
43	D
44	A
45	B
46	D
47	A
48	C
49	C
50	D
51	B
52	A
53	A
54	C
55	C
56	C
57	C

58	C
59	A
60	D
61	C
62	C
63	C
64	C
65	B
66	B
67	A
68	D
69	D
70	B
71	C
72	C
73	C
74	B
75	B
76	D
77	A
78	A
79	B
80	B
81	A
82	C
83	A
84	C
85	B

UNIT-II

1. The Lebesgue integral is an extension of the:

   a. Riemann integral
   b. Fourier integral
   c. Stieltjes integral
   d. Improper integral

   Hint: Think about which integral it builds upon.

2. The Lebesgue integral was developed to address issues related to:

   a. Convergence of series
   b. Integration of discontinuous functions
   c. Integration of complex functions
   d. Approximation of integrals

   Hint: Reflect on the difficulties encountered in traditional integration methods.

3. The Riemann integral requires a function to be:

   a. Continuous
   b. Differentiable
   c. Bounded
   d. Integrable

   Hint: Consider the requirement for a function to be integrable in the Riemann sense.

4. The Riemann integral approximates the area under a curve using:

a. Sums of rectangles
b. Sums of triangles
c. Sums of trapezoids
d. Sums of circles

Hint: Reflect on the method used in the Riemann integral for approximation.

5. The Lebesgue integral can handle functions that are not:

a. Continuous
b. Bounded
c. Differentiable
d. Integrable

Hint: Think about the kind of functions the Lebesgue integral can handle.

6. The Riemann integral is defined using:

a. Upper and lower Darboux sums
b. Upper and lower Riemann sums
c. Upper and lower Stieltjes sums
d. Upper and lower Lebesgue sums

Hint: Recall the specific sums used in the definition of the Riemann integral.

7. The Lebesgue integral allows integration of functions that are:

a. Piecewise continuous
b. Absolutely continuous
c. Uniformly continuous
d. Pointwise continuous

Hint: Reflect on the types of functions handled by the Lebesgue integral.

8. Which integral gives more flexibility in handling the limit of sequences of functions?

a. Riemann integral
b. Lebesgue integral
c. Stieltjes integral
d. Cauchy integral

Hint: Consider the property regarding the limit of functions.

9. The Riemann integral is based on the partition of an interval into:

a. Open sets
b. Closed sets
c. Disjoint sets
d. Subintervals

Hint: Think about how the interval is divided in the Riemann integral.

10. The Lebesgue integral can handle functions that are not Riemann integrable due to issues with:

a. Convergence
b. Discontinuity
c. Divergence
d. Unboundedness

Hint: Reflect on the problem encountered in Riemann integrability.

11. The Lebesgue integral is particularly useful for functions with:

a. Pointwise continuity
b. Pointwise discontinuity
c. Uniform continuity
d. Absolute continuity

Hint: Consider the kind of continuity relevant to the Lebesgue integral.

12. The Lebesgue integral allows integration of functions that are not bounded because of its treatment of:

a. Limits
b. Discontinuities
c. Oscillations

d. Monotonicity

Hint: Think about the handling of unbounded functions in the Lebesgue integral.

13. The Riemann integral can be calculated using:

a. The partition of the domain
b. The partition of the range
c. The partition of the graph
d. The partition of the derivative

Hint: Reflect on the method used to calculate the Riemann integral.

14. The Lebesgue integral is defined in terms of:

a. Upper and lower sums
b. Measures and measurable sets
c. Partitions and subintervals
d. Infimum and supremum

Hint: Reflect on the components involved in defining the Lebesgue integral.

15. The Riemann integral considers the values of a function on:

a. All points in an interval
b. A countable set of points in an interval
c. A dense set of points in an interval

d. An open set of points in an interval

Hint: Consider the points considered for the Riemann integral.

16. The Lebesgue integral's domain of integration is specified by:

a. Partitions
b. Subintervals
c. Measures
d. Limits

Hint: Think about the domain over which the Lebesgue integral operates.

17. The Lebesgue integral handles functions that might not be integrable in the Riemann sense due to their behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of boundedness

Hint: Consider the problematic points for Riemann integrability.

18. The Riemann integral is based on the concept of:

a. Outer measures
b. Measures of disjoint sets
c. Partitioning and summation

d. Convergence of series

Hint: Consider the fundamental concept employed in the Riemann integral.

19. The Lebesgue integral is more powerful in handling the limit of sequences of functions because of its treatment of:

a. Oscillations
b. Continuity
c. Differentiability
d. Integrability

Hint: Think about the issue addressed by the Lebesgue integral in the context of limits.

20. The Lebesgue integral allows the integration of functions that may not be Riemann integrable due to their behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of unboundedness

Hint: Reflect on the problematic points for Riemann integrability.

21. The Riemann integral is defined over intervals using:

a. Sums of rectangles
b. Sums of trapezoids
c. Sums of circles
d. Sums of triangles

Hint: Consider the method used to approximate area in the Riemann integral.

22. The Lebesgue integral is well-defined for functions that are:

a. Uniformly continuous
b. Riemann integrable
c. Absolutely continuous
d. Bounded

Hint: Think about the kind of functions well-handled by the Lebesgue integral.

23. The Riemann integral focuses on the behavior of functions at:

a. Discontinuities
b. Oscillations
c. Unboundedness
d. Points of differentiability

Hint: Reflect on the aspect considered in the Riemann integral.

24. The Lebesgue integral is particularly useful for functions with:

a. Pointwise continuity
b. Pointwise discontinuity
c. Uniform continuity
d. Absolute continuity

Hint: Consider the type of function behavior addressed by the Lebesgue integral.

25. The Riemann integral can be applied to functions that are not necessarily:

a. Bounded
b. Continuous
c. Monotonic
d. Discontinuous

Hint: Reflect on the properties required for functions in Riemann integration.

26. The Lebesgue integral can handle functions that are not Riemann integrable due to their behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of boundedness

Hint: Reflect on the problematic points for Riemann integrability.

27. The Riemann integral primarily focuses on the behavior of functions concerning their:

a. Continuity
b. Discontinuity
c. Monotonicity
d. Uniformity

Hint: Reflect on the main aspect addressed by the Riemann integral.

28. The Lebesgue integral allows the integration of functions that are not Riemann integrable due to their behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of boundedness

Hint: Consider the problematic points for Riemann integrability.

29. The Riemann integral is concerned with the behavior of functions at:

a. Discontinuities
b. Oscillations
c. Unboundedness
d. Points of differentiability

Hint: Reflect on the aspect considered in the Riemann integral.

30. The Lebesgue integral handles functions that are not Riemann integrable due to their behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of unboundedness

Hint: Consider the problematic points for Riemann integrability.

31. The Lebesgue integral of a bounded function over a set of finite measure is always:

a. Finite
b. Infinite
c. Zero
d. Undefined

Hint: Consider the property related to the measure of the set.

32. For a bounded function on a set of finite measure, the Lebesgue integral corresponds to the:

a. Upper Riemann sum
b. Lower Riemann sum
c. Total variation
d. Mean value

Hint: Reflect on the relationship between the Lebesgue integral and approximation sums.

33. The Lebesgue integral of a bounded function on a set of finite measure corresponds to the Riemann integral if the function is:

a. Continuous
b. Differentiable
c. Monotonic
d. Piecewise constant

Hint: Consider the kind of functions for which the Lebesgue integral and the Riemann integral align.

34. The Lebesgue integral of a bounded function over a set of finite measure is preserved under:

a. Convergence in measure
b. Pointwise convergence
c. Uniform convergence
d. Convergence in distribution

Hint: Consider the kind of convergence that maintains the integral value.

35. The Lebesgue integral of a bounded function on a set of finite measure is affected by the function's behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of boundedness

Hint: Reflect on the problematic points for Lebesgue integrability.

36. The Lebesgue integral of a bounded function over a set of finite measure corresponds to the:

a. Infimum of the upper Riemann sums
b. Supremum of the lower Riemann sums
c. Difference between upper and lower Riemann sums
d. Sum of upper and lower Riemann sums

Hint: Consider the relationship between the Lebesgue integral and Riemann sums.

37. The Lebesgue integral of a bounded function on a set of finite measure is well-defined because of its property of:

a. Additivity
b. Linearity
c. Absolute continuity
d. Monotonicity

Hint: Reflect on the property that ensures well-definedness of the Lebesgue integral.

38. The Lebesgue integral of a bounded function over a set of finite measure corresponds to the Riemann integral for functions that are:

a. Monotonic
b. Continuous

c. Discontinuous
d. Piecewise constant

Hint: Consider the kind of functions for which the Lebesgue integral aligns with the Riemann integral.

39. The Lebesgue integral of a bounded function over a set of finite measure is invariant under:

a. Change of variables
b. Change of limits
c. Change of measure
d. Change of domain

Hint: Reflect on the property that remains unchanged during certain transformations.

40. The Lebesgue integral of a bounded function on a set of finite measure can be computed using:

a. Upper and lower sums
b. Partition of the range
c. Measures of disjoint sets
d. Infimum and supremum

Hint: Reflect on the method used to compute Lebesgue integrals.

41. The integral of a nonnegative function is always:

a. Nonnegative
b. Nonpositive

c. Positive
d. Negative

Hint: Consider the property related to the sign of the integral for nonnegative functions.

42. The integral of a nonnegative function corresponds to the area under the curve when the function is:

a. Monotonic
b. Continuous
c. Differentiable
d. Bounded

Hint: Think about the geometrical interpretation of the integral for nonnegative functions.

43. The integral of a nonnegative function over a set is affected by the function's behavior at:

A) Points of continuity

B) Points of differentiability

C) Points of discontinuity

D) Points of boundedness

Hint: Reflect on the problematic points for integrability of nonnegative functions.

44. The integral of a nonnegative function is well-defined because of its property of:

a. Additivity
b. Linearity
c. Absolute continuity
d. Monotonicity

Hint: Consider the property that ensures well-definedness of the integral for nonnegative functions.

45. The integral of a nonnegative function corresponds to the:

a. Supremum of upper Riemann sums
b. Infimum of lower Riemann sums
c. Difference between upper and lower Riemann sums
d. Sum of upper and lower Riemann sums

Hint: Reflect on the relationship between the integral and Riemann sums for nonnegative functions.

46. The integral of a nonnegative function is preserved under:

a. Convergence in measure
b. Pointwise convergence
c. Uniform convergence
d. Convergence in distribution

Hint: Consider the kind of convergence that maintains the integral value for nonnegative functions.

47. The integral of a nonnegative function corresponds to the total area under the curve when the function is:

a. Monotonic
b. Continuous
c. Discontinuous
d. Unbounded

Hint: Think about the geometrical interpretation of the integral for nonnegative functions.

48. The integral of a nonnegative function is invariant under:

a. Change of variables
b. Change of limits
c. Change of measure
d. Change of domain

Hint: Reflect on the property that remains unchanged during certain transformations.

49. The integral of a nonnegative function can be computed using:

a. Upper and lower sums
b. Partition of the range
c. Measures of disjoint sets
d. Infimum and supremum

Hint: Reflect on the method used to compute integrals for nonnegative functions.

50. The integral of a non-negative function is always greater than or equal to:

a. Zero
b. One
c. Negative one
d. Undefined

Hint: Consider the property related to the sign of the integral for nonnegative functions.

51. The Lebesgue integral of a function over a set is defined in terms of:

a. Upper and lower sums
b. Measures and measurable sets
c. Partitions and subintervals
d. Infimum and supremum

Hint: Reflect on the components involved in defining the general Lebesgue integral.

52. The Lebesgue integral of a function over a set corresponds to the Riemann integral if the function is:

a. Continuous
b. Differentiable
c. Monotonic
d. Piecewise constant

Hint: Consider the kind of functions for which the Lebesgue integral and the Riemann integral align.

53. The Lebesgue integral of a function is affected by the function's behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity

d. Points of boundedness

Hint: Reflect on the problematic points for Lebesgue integrability.

54. The Lebesgue integral of a function corresponds to the:

a. Infimum of the upper Riemann sums
b. Supremum of the lower Riemann sums
c. Difference between upper and lower Riemann sums
d. Sum of upper and lower Riemann sums

Hint: Consider the relationship between the Lebesgue integral and Riemann sums.

a. 55. The Lebesgue integral of a function is well-defined because of its property of:
    a. Additivity
    b. Linearity
    c. Absolute continuity
    d. Monotonicity

Hint: Reflect on the property that ensures well-definedness of the Lebesgue integral.

56. The Lebesgue integral of a function corresponds to the Riemann integral for functions that are:

a. Monotonic

b. Continuous
c. Discontinuous
d. Piecewise constant

Hint: Consider the kind of functions for which the Lebesgue integral aligns with the Riemann integral.

57. The Lebesgue integral of a function is invariant under:

a. Change of variables
b. Change of limits
c. Change of measure
d. Change of domain

Hint: Reflect on the property that remains unchanged during certain transformations.

58. The Lebesgue integral of a function can be computed using:

a. Upper and lower sums
b. Partition of the range
c. Measures of disjoint sets
d. Infimum and supremum

Hint: Reflect on the method used to compute Lebesgue integrals.

59. The Lebesgue integral of a function is affected by the function's behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of unboundedness

Hint: Reflect on the problematic points for Lebesgue integrability.

60. The Lebesgue integral of a function corresponds to the Riemann integral for functions that are:

a. Monotonic
b. Continuous
c. Discontinuous
d. Bounded

Hint: Consider the kind of functions for which the Lebesgue integral aligns with the Riemann integral.

61. The Lebesgue integral of a function is preserved under:

a. Convergence in measure
b. Pointwise convergence
c. Uniform convergence
d. Convergence in distribution

Hint: Consider the kind of convergence that maintains the integral value.

Answer: B) Pointwise convergence

62. The Lebesgue integral of a function is well-defined because of its property of:

a. Additivity
b. Linearity
c. Absolute continuity
d. Monotonicity

Hint: Reflect on the property that ensures well-definedness of the Lebesgue integral.

63. The Lebesgue integral of a function over a set corresponds to the:

a. Supremum of upper Riemann sums
b. Infimum of lower Riemann sums
c. Difference between upper and lower Riemann sums
d. Sum of upper and lower Riemann sums

Hint: Reflect on the relationship between the integral and Riemann sums.

64. The Lebesgue integral of a function is well-defined for functions that are:

a. Uniformly continuous
b. Riemann integrable
c. Absolutely continuous
d. Bounded

Hint: Consider the kind of functions well-handled by the Lebesgue integral.

65. The Lebesgue integral of a function is affected by the function's behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of unboundedness

Hint: Reflect on the problematic points for Lebesgue integrability.

66. The Lebesgue integral of a function can be computed using:

a. Upper and lower sums
b. Partition of the range
c. Measures of disjoint sets
d. Infimum and supremum

Hint: Reflect on the method used to compute Lebesgue integrals.

67. The Lebesgue integral of a function corresponds to the:

a. Infimum of the upper Riemann sums
b. Supremum of the lower Riemann sums

c. Difference between upper and lower Riemann sums
d. Sum of upper and lower Riemann sums

Hint: Reflect on the relationship between the integral and Riemann sums.

68. The Lebesgue integral of a function is well-defined because of its property of:

a. Additivity
b. Linearity
c. Absolute continuity
d. Monotonicity

Hint: Reflect on the property that ensures well-definedness of the Lebesgue integral.

69. The Lebesgue integral of a function is affected by the function's behavior at:

a. Points of continuity
b. Points of differentiability
c. Points of discontinuity
d. Points of boundedness

Hint: Reflect on the problematic points for Lebesgue integral

70. The Riemann integral is primarily used for:

a. Integrating continuous functions

b. Integrating discontinuous functions
c. Integrating on bounded intervals
d. Integrating on unbounded intervals

Hint: Consider the domain and properties of functions suitable for the Riemann integral.

71. The Riemann integral requires a function to be:

a. Continuous on the interval of integration
b. Monotonic on the interval of integration
c. Differentiable on the interval of integration
d. Piecewise continuous on the interval of integration

Hint: Recall the specific conditions necessary for a function to be Riemann integrable.

72. Lebesgue's generalization of the integral extends integration to functions that are:

a. Continuous
b. Differentiable
c. Discontinuous
d. Piecewise continuous

Hint: Think about the extension of integration to a broader class of functions.

73. The Lebesgue integral is defined for functions on sets that are:

a. Bounded
b. Countable
c. Measurable
d. Compact

Hint: Consider the relationship between the Lebesgue integral and the properties of sets.

74. A function that is Riemann integrable is always:

a. Lebesgue integrable
b. Lebesgue measurable
c. Continuous
d. Monotonic

Hint: Consider the relationship between Riemann and Lebesgue integrability.

75. The Riemann integral is based on:

a. Upper and lower Darboux sums
b. Upper and lower Riemann sums
c. Upper and lower limits
d. Upper and lower bounds

Hint: Recall the fundamental components used to compute the Riemann integral.

76. The Lebesgue integral handles discontinuities in functions by:

a. Ignoring them
b. Approximating them

c. Measuring their size
d. Disregarding their contribution to the integral

Hint: Consider how the Lebesgue integral deals with discontinuities in functions.

77. For a function to be Lebesgue integrable, it must be:

a. Bounded
b. Continuous
c. Uniformly continuous
d. Essentially bounded

Hint: Focus on the properties required for Lebesgue integrability.

78. The Lebesgue integral is well-suited for functions with:

a. Discontinuities
b. Polynomial behavior
c. Exponential growth
d. Finite limits

Hint: Consider the types of functions that benefit from the Lebesgue integral.

79. One of the advantages of the Lebesgue integral over the Riemann integral is its ability to:

a. Handle unbounded intervals

b. Handle only continuous functions
c. Calculate definite integrals easily
d. Avoid discontinuities

Hint: Consider the limitations of the Riemann integral that the Lebesgue integral addresses.

80. The Riemann integral can be extended to non-Riemann integrable functions using:

a. Cauchy's criterion
b. Lebesgue's criterion
c. Monotone convergence theorem
d. Fatou's lemma

Hint: Think about the method that allows extension beyond Riemann integrability.

81. Lebesgue's integration is especially useful for functions that:

a. Are continuous on closed intervals
b. Have point discontinuities
c. Are bounded on open intervals
d. Are monotonic

Hint: Consider the types of functions where Lebesgue integration provides advantages.

82. The Lebesgue integral of a non-negative function can be approximated using:

a. Riemann sums
b. Upper and lower Darboux sums
c. Simple functions
d. Cesàro sums

Hint: Think about the methods used to approximate integrals in Lebesgue integration.

83. The concept of measurability is closely related to:

a. Riemann sums
b. Riemann integrability
c. Lebesgue sets
d. Compact sets

Hint: Consider the underlying concepts associated with measure theory.

84. The Lebesgue integral's definition involves integration with respect to:

a. Riemann sums
b. Measure spaces
c. Riemann measures
d. Lower sums

Hint: Recall the foundation upon which the Lebesgue integral is built.

ANSWERS

S.NO	OPTIONS
1	A
2	B
3	C
4	A
5	A
6	B
7	B
8	B
9	D
10	B
11	B
12	C
13	A
14	B
15	A
16	C
17	C
18	C
19	A
20	C
21	A
22	C
23	A
24	B
25	B
26	C
27	A
28	C

29	A
30	C
31	A
32	C
33	D
34	C
35	C
36	C
37	A
38	D
39	C
40	A
41	A
42	B
43	C
44	A
45	D
46	B
47	C
48	C
49	A
50	A
51	B
52	D
53	C
54	C
55	A
56	D
57	C
58	A
59	C

60	C
61	B
62	A
63	D
64	C
65	C
66	C
67	A
68	C
69	C
70	C
71	D
72	D
73	C
74	A
75	B
76	C
77	D
78	A
79	A
80	B
81	B
82	C
83	B
84	B

UNIT-III

1. Which of the following statements is true about monotone functions?

a. They can only be increasing.
b. They can only be decreasing.
c. They can either be increasing or decreasing.
d. They must be both increasing and decreasing.

Hint: Consider the nature of monotone functions regarding their behavior.

2. For a function to be strictly increasing on an interval, which condition must hold true?

a. The derivative is always positive on that interval.
b. The derivative is always negative on that interval.
c. The derivative is either positive or zero on that interval.
d. The derivative is either negative or zero on that interval.

Hint: Think about the behavior of the derivative for a strictly increasing function.

3. Which of the following functions is NOT monotone on its entire domain?

a. f(x) = 3x + 2
b. g(x) = e^x
c. h(x) = x^3 + 2x
d. k(x) = {1/x}

e. Hint: Examine the behavior of each function and its derivatives.

4. What can be said about the derivative of a strictly decreasing function?

a. The derivative is always negative.
b. The derivative is always positive.
c. The derivative can be positive or negative.
d. The derivative is always zero.

Hint: Think about the slope of a decreasing function.

5. Which of the following is true for a function that is monotone on an interval [a, b]?

a. It can have only a finite number of extrema in that interval.
b. It cannot have any critical points in that interval.
c. It can have at most countably many points of discontinuity in that interval.
d. It must be continuous on that interval.

Hint: Consider the relationship between monotonicity and other properties of functions.

6. If a function is monotone on an interval [a, b], what can be inferred about its integrability on that interval?

a. It is always Riemann integrable.
b. It is always Lebesgue integrable.
c. It may or may not be Riemann integrable.
d. It may or may not be Lebesgue integrable.

Hint: Think about the properties of monotone functions and integrability.

7. Which of the following functions is monotone on its entire domain?

a. f(x) = x^2 + 3x + 5
b. g(x) = $\sqrt{x}$
c. h(x) = sin(x)
d. k(x) = e^{-x}

Hint: Analyze the behavior of each function over its domain.

8. For a differentiable function that is strictly increasing on an interval, what can be said about its derivative?

a. The derivative is always positive.
b. The derivative is always negative.
c. The derivative is always zero.
d. The derivative can be positive or zero.

Hint: Consider the behavior of a strictly increasing function.

9. Which of the following statements about the derivative of a monotone function is true?

a. It can have removable discontinuities.
b. It can have jump discontinuities.
c. It cannot have any type of discontinuity.
d. It can have infinite discontinuities.

Hint: Think about the nature of monotone functions and their derivatives.

10. If a function is monotone on its entire domain, what can be concluded about its limits at infinity?

a. Both limits at infinity exist.
b. One limit at infinity exists.
c. None of the limits at infinity exist.
d. The limits at infinity oscillate.

Hint: Consider the behavior of monotone functions at the extremes of their domains.

11. Which of the following statements is true for a function that is monotone and has a finite derivative almost everywhere?

a. It is necessarily continuous.

b. It can have countably many points of discontinuity.
c. Its derivative is also monotone.
d. It cannot have any critical points.

Hint: Think about the relationship between the derivative and the behavior of the function.

12. Which type of monotonicity implies that the function is either strictly increasing or strictly decreasing but not both on an interval?

a. Weak monotonicity
b. Strong monotonicity
c. Absolute monotonicity
d. Strict monotonicity

Hint: Consider the characteristics of different types of monotonicity.

13. For a function that is monotone on an interval, what can be said about its set of discontinuities?

a. It can have only a finite number of discontinuities.
b. It must have countably many discontinuities.
c. The set of discontinuities can be uncountable.
d. It cannot have any discontinuities.

Hint: Consider the nature of discontinuities for monotone functions.

14. What can be said about a monotone function's behavior at a point of discontinuity?

a. It must have a removable discontinuity.
b. It must have a jump discontinuity.
c. It cannot have any type of discontinuity.
d. It must have a limit from both sides at that point.

Hint: Consider the behavior of monotone functions around points of discontinuity.

15. Which of the following functions is monotone but not differentiable at a point in its domain?

a. f(x) = |x|
b. g(x) = x^(1/3)
c. \(h(x) = 1/x
d. k(x) = sin(x)

Hint: Examine the behavior of each function regarding monotonicity and differentiability.

16. For a strictly decreasing function, what can be inferred about its behavior at points of discontinuity?

a. It must have a removable discontinuity.
b. It must have a jump discontinuity.
c. It cannot have any type of discontinuity.
d. It must have a limit from both sides at that point.

Hint: Think about the characteristics of strictly decreasing functions.

17. Which of the following statements is true for a monotone function on a closed interval [a, b]?

a. It can have at most one maximum and one minimum in that interval.
b. It must have at least one maximum and one minimum in that interval.
c. It cannot have any extrema in that interval.
d. It can have countably many maxima and minima in that interval.

Hint: Consider the behavior of monotone functions regarding extrema.

18. What is the relationship between a function being monotone and its inverse function?

a. The inverse function is also monotone.
b. The inverse function may or may not be monotone.
c. The inverse function is always strictly increasing.
d. The inverse function is always strictly decreasing.

Hint: Consider the relationship between the behavior of a function and its inverse.

19. Which of the following is true for a function that is strictly increasing on its entire domain?

a. Its derivative is always positive.
b. Its derivative is always negative.
c. Its derivative is always zero.
d. Its derivative is always bounded.

Hint: Think about the behavior of a strictly increasing function and its derivative.

20. What can be said about the continuity of a monotone function on a closed interval?

a. It must be continuous at every point in the interval.
b. It can have countably many points of discontinuity in the interval.
c. It must be discontinuous at atleast one point in the interval.
d. It must be discontinuous at all points in the interval.

Hint: Consider the relationship between monotonicity and continuity.

21. Which of the following is true regarding the Lebesgue integral of a bounded function over a set of finite measure?

a. It always exists.

b. It exists only for continuous functions.
c. It exists for measurable functions.
d. It exists if the function is unbounded.

Hint: Consider the basic properties of Lebesgue integrals and what conditions are necessary for the existence of the integral over a set of finite measure.

22. For a bounded function f over a set of finite measure, which theorem provides a condition for its integrability in terms of its measurability?

a. Monotone Convergence Theorem
b. Dominated Convergence Theorem
c. Fatou's Lemma
d. Radon-Nikodym Theorem

Hint:Think about the theorems that provide conditions for the integrability of measurable functions over a set of finite measure.

23. Consider a bounded function f defined on a set of finite measure. Which property ensures that the Lebesgue integral of f can be computed by integrating its pointwise limit?

a. Continuity of f
b. Uniform convergence of f
c. Pointwise convergence of f
d. Monotonicity of f

Hint: Think about the convergence properties that allow changing the order of integration and limit.

24. What is the relationship between Riemann integrability and Lebesgue integrability for bounded functions over a set of finite measure?

a. Every Riemann integrable function is Lebesgue integrable.
b. Every Lebesgue integrable function is Riemann integrable.
c. They are equivalent for bounded functions.
d. There is no relation between them.

Hint: Consider the conditions and properties of Riemann integrability and Lebesgue integrability for bounded functions.

25. For a bounded function f over a set of finite measure, if f is integrable, what can be said about the set of points where f is not continuous?

a. It must have measure zero.
b. It must be dense in the set.
c. It is not related to the integrability of f .
d. It must have positive measure.

Hint: Think about the relationship between the integrability of a function and the points where it might not be continuous.

26. What is the significance of a nonnegative function in integration theory?

   a. It simplifies the integration process.
   b. It always results in a finite integral.
   c. It always converges.
   d. It can only be integrated using the Riemann integral.

Hint: Consider the behavior of a nonnegative function with respect to integration and convergence.

Answer: b) It always results in a finite integral.

27. Which of the following statements is true regarding the integral of a nonnegative function?

   a. It can be negative.
   b. It can be infinite.
   c. It is always zero.
   d. It is equal to the supremu

Hint: Focus on the properties of nonnegative functions and their relation to the integral.

28. For a nonnegative function, which theorem is commonly used to interchange the integral and the limit?

   a. Monotone Convergence Theorem
   b. Dominated Convergence Theorem
   c. Fatou's Lemma
   d. Radon-Nikodym Theorem

Hint: Consider the theorems related to the convergence of integrals for nonnegative functions.

29. Which of the following integrals best represents the integral of a nonnegative function *f* over a set *E*?

a. $\int_E f(x)dx$

b. $\int_E |f(x)|\, dx$

c. $\int_E \max(f(x), 0)\, dx$

d. $\int_E \min(f(x), 0)\, dx$

Hint: Consider the nature of a nonnegative function and how its integral is formulated.

30. For a nonnegative function *f* and a measurable set *E*, which property regarding the integral holds true?

a. If *f* is unbounded, the integral over *E* does not exist.
b. The integral over the empty set is always zero.
c. The integral over any set *E* is always finite.
d. The integral of a nonnegative function is always equal to the supremum of its values on *E*.

Hint: Think about the basic properties of integrals for nonnegative functions and their relation to sets.

31. What is the definition of the integral for a nonnegative function f over a set E ?

a. $\int_E f(x)dx$=sup{ Lower sums of f}

b. $\int_E f(x)dx$=inf{ Upper sums of f}

c. $\int_E f(x)dx$=sup{ Upper sums of f}

d. $\int_E f(x)dx$=inf{ Lower sums of f}

Hint: Consider the relationship between lower and upper sums and their significance in defining the integral of a function.

32. Which property is true for the integral of a nonnegative function over a set?

a. It is always finite.

b. It is always zero.

c. It is always equal to the supremum of the function.

d. It is always equal to the infimum of the function

Hint: Think about the behavior of nonnegative functions with respect to the integral.

33. What does the Monotone Convergence Theorem state about the integral of a sequence of nonnegative functions?

a. If the sequence is bounded, then the integral is finite.

b. If the sequence is decreasing, then the integral converges to zero.

c. If the sequence is increasing, then the integral of the limit is the limit of the integrals.

d. The integral of the limit is always zero.

Hint: Consider the convergence properties of nonnegative functions and their relation to the integral.

34. For a nonnegative function f , which theorem allows the interchange of limit and integral under certain conditions?

a. Fatou's Lemma
b. Dominated Convergence Theorem
c. Monotone Convergence Theorem
d. Fubini's Theorem

Hint: Think about the theorems that enable the interchange of limit and integral for nonnegative functions.

35. Which of the following is a property of the integral of a nonnegative function over an empty set?

a. It is always infinite.
b. It is always zero.
c. It is undefined.
d. It is equal to the supremum of the function.

Hint: Consider the definition of the integral and its behavior over an empty set.

36. Which of the following statements correctly defines the Lebesgue integral for a function f on a measurable set E ?

a. It is defined as the limit of Riemann sums as the number of subdivisions approaches infinity.

b. It is defined as the limit of upper and lower sums as the mesh of partitions approaches zero.
c. It is defined as the supremum of approximating step functions.
d. It is defined as the limit of the integral of simple functions converging to f .

Hint:Think about how the Lebesgue integral is constructed using approximating functions

37. Which of the following functions is Lebesgue integrable on a measurable set E ?

a. f(x) = (1/x) on E = (0, 1)
b. f(x) = sin(x) on E = [0, π]
c. f(x) = $\frac{1}{\sqrt{x}}$ on E = [0, 1]
d. f(x) = $\frac{1}{x^2}$ on E = [1, ∞]

Hint: Consider the integrability conditions for Lebesgue integrals and the properties of the functions given.

38. Which theorem guarantees the existence of the Lebesgue integral for a measurable function on a finite measure space?

a. Monotone Convergence Theorem
b. Dominated Convergence Theorem
c. Fatou's Lemma
d. Radon-Nikodym Theorem

Hint: Think about the theorems that ensure the existence of the Lebesgue integral under certain conditions.

39. For which of the following sets does the Lebesgue integral coincide with the Riemann integral for a bounded function?

a. Open interval
b. Closed interval
c. Discrete set
d. Infinite interval

Hint: Consider the relationship between the Lebesgue and Riemann integrals for different types of sets.

40. Which property ensures the linearity of the Lebesgue integral?

a. Dominated Convergence Theorem
b. Monotone Convergence Theorem
c. Linearity of integrals for simple functions
d. Fatou's Lemma

Hint: Think about the properties that allow the Lebesgue integral to be linear.

41. For a non-negative function f on a measurable set E, if f(x) = 0 almost everywhere on E, what can be said about the Lebesgue integral of f over E?

a. It is always zero.
b. It is always infinity.
c. It may not exist.
d. It is equal to the supremum of \(f\) on \(E\).

Hint: Consider the definition and properties of the Lebesgue integral for non-negative functions.

42. Which theorem is used to interchange the limit and the Lebesgue integral for a sequence of measurable functions?

a. Monotone Convergence Theorem
b. Dominated Convergence Theorem
c. Fatou's Lemma
d. Bounded Convergence Theorem

Hint: Think about the theorem that specifies the conditions for exchanging limit and integral.

43. What is a necessary condition for a function f to be Lebesgue integrable on a measurable set E?

a. The function must be continuous on E.
b. The function must be bounded on E.
c. The function must be Riemann integrable on E.
d. The function must be differentiable on E.

Hint: Consider the fundamental properties required for a function to be Lebesgue integrable.

44. Which property holds true for the integral of a measurable function f over an empty set Emptyset?

a. It is always zero.
b. It is always infinity.
c. It does not exist.
d. It is equal to the supremum of f on emptyset.

Hint: Think about the integral of a function over an empty set and its relationship with measure.

45. Monotone functions are:
    a. Always differentiable
    b. Always continuous
    c. Either differentiable almost everywhere or have a countable number of discontinuities
    d. Always bounded

Hint: Consider the characteristics of monotone functions regarding their differentiability.

46. A function that is monotone on an interval can have:
    a. Infinitely many discontinuities
    b. Only one discontinuity
    c. At most countably many discontinuities
    d. No discontinuities

Hint: Think about the properties of monotone functions concerning the number of possible discontinuities.

47. Absolute continuity of a function implies:
    a. The function is continuous
    b. The function has a derivative almost everywhere
    c. The function has a continuous derivative
    d. The function satisfies the Lipschitz condition

Hint: Consider the conditions required for absolute continuity and its relation to derivatives.

48. A function that is absolutely continuous on an interval is also:

a. Monotone
b. Differentiable
c. Continuous
d. Unbounded

Hint: Consider the properties shared by functions that are absolutely continuous.

49. Absolute continuity implies that the function has:

a. A continuous derivative
b. A bounded derivative
c. A derivative almost everywhere
d. A Lipschitz continuous derivative

Hint: Consider the implications of absolute continuity on the derivative of a function.

50. The derivative of an absolutely continuous function:

a. Exists at every point
b. Exists almost everywhere
c. Does not exist
d. Is unbounded

Hint: Consider the characteristics of the derivative of an absolutely continuous function.

51. If a function is absolutely continuous, then it is also:

a. Monotone
b. Lipschitz continuous
c. Differentiable

d. Uniformly continuous

Hint: Think about the additional properties associated with absolute continuity.

52. A function that is absolutely continuous on a closed interval is necessarily:

a. Bounded
b. Differentiable everywhere
c. Monotone
d. Continuous

Hint: Consider the properties of functions that are absolutely continuous on a closed interval.

53. Absolute continuity is a stronger property than:

a. Continuity
b. Differentiability
c. Monotonicity
d. Uniform continuity

Hint: Consider the hierarchy of properties in functions and their relationships.

54. If a function is absolutely continuous, then it must be:

a. Riemann integrable
b. Lebesgue integrable
c. Improperly integrable
d. Continuous

Hint: Consider the integrability properties related to absolute continuity.

55. Functions that are absolutely continuous on an interval are necessarily:

a. Absolutely integrable
b. Riemann integrable
c. Monotone
d. Continuous

Hint: Think about the relationship between absolute continuity and integrability.

56. A function that is absolutely continuous on an interval has:

a. A continuous derivative
b. A bounded derivative
c. A derivative almost everywhere
d. A continuous second derivative

Hint: Recall the properties of the derivative in relation to absolute continuity.

57. Absolute continuity of a function implies its:

a. Continuity
b. Uniform continuity
c. Monotonicity
d. Differentiability almost everywhere

Hint: Consider the implications of absolute continuity on various properties of a function.

58. Functions that are absolutely continuous have a special property concerning:

a. The size of their discontinuities
b. The behavior of their derivatives
c. The frequency of their oscillations
d. The limits of their integrals

Hint: Think about the particular aspect that characterizes functions with absolute continuity.

59. The Cantor-Lebesgue function is an example of a function that is:

a. Absolutely continuous
b. Monotone but not absolutely continuous
c. Absolutely continuous but not monotone
d. Neither absolutely continuous nor monotone

Hint: Consider the properties of the Cantor-Lebesgue function concerning absolute continuity and monotonicity.

60. Absolute continuity of a function implies:

a. Differentiability
b. Continuity
c. Monotonicity
d. Boundedness

Hint: Consider the relationship between absolute continuity and other properties of functions.

61. A function that is absolutely continuous on an interval is also:

a. Bounded
b. Monotone

c. Discontinuous
d. Non-differentiable

Hint: Think about additional properties shared by functions that are absolutely continuous.

Answer: b) Monotone

62. If a function is absolutely continuous on a closed interval, then it is necessarily:

a. Continuous
b. Differentiable almost everywhere
c. Monotone
d. Discontinuous

Hint: Consider the consequences of absolute continuity on the properties of a function.

63. Absolute continuity of a function on an interval implies:

a. The function is continuous
b. The function is uniformly continuous
c. The function has a derivative almost everywhere
d. The function is monotone

Hint: Consider the characteristics of functions that are absolutely continuous.

64. The derivative of an absolutely continuous function:

a. Exists everywhere
b. Exists almost everywhere
c. Does not exist

d. Is bounded

Hint: Think about the nature of the derivative of an absolutely continuous function.

65. A function that is absolutely continuous on an interval is also:

a. Absolutely integrable
b. Differentiable everywhere
c. Continuous
d. Riemann integrable

Hint: Consider the integrability properties associated with absolute continuity.

66. If a function is absolutely continuous on an interval, then it is also:

a. Absolutely continuous on any subinterval
b. Discontinuous on any subinterval
c. Not differentiable on any subinterval
d. Monotone on any subinterval

Hint: Think about the properties of absolute continuity on subintervals.

67. Absolute continuity is a stronger property than:

a. Uniform continuity
b. Continuity
c. Differentiability
d. Monotonicity

Hint: Consider the hierarchy of properties in functions and their relationships.

68. A function that is absolutely continuous on a closed interval is necessarily:

a. Continuous
b. Uniformly continuous
c. Monotone
d. Discontinuous

Hint: Think about the additional properties associated with absolute continuity.

69. The Cantor-Lebesgue function is an example of a function that is:

a. Absolutely continuous
b. Monotone but not absolutely continuous
c. Absolutely continuous but not monotone
d. Neither absolutely continuous nor monotone

Hint: Consider the characteristics of the Cantor-Lebesgue function concerning absolute continuity and monotonicity

70. The derivative of a function that is absolutely continuous almost everywhere is:

a. Bounded
b. Continuous
c. Discontinuous
d. Unbounded

Hint: Consider the relationship between the derivative and absolute continuity.

71. If a function is absolutely continuous on a closed interval, then it is also:

a. Absolutely integrable
b. Riemann integrable
c. Lebesgue integrable
d. Improperly integrable

Hint: Think about the integrability properties related to absolute continuity.

72. A function that is absolutely continuous has a derivative that satisfies which property?
a. It is continuous
b. It is bounded
c. It is monotonic
d. It is discontinuous

Hint: Consider the characteristics of the derivative of an absolutely continuous function.

73. A function that is absolutely continuous has a special property concerning:
a. The size of its discontinuities
b. The behavior of its derivatives
c. The frequency of its oscillations
d. The limits of its integral

Hint: Think about the specific aspect characterizing functions with absolute continuity.

74. Absolute continuity of a function implies its:
a. Differentiability
b. Uniform continuity

c. Monotonicity
d. Continuity

Hint: Consider the implications of absolute continuity on various properties of a function.

ANSWERS

S.NO	OPTIONS
1	C
2	A
3	D
4	A
5	C
6	A
7	D
8	A
9	D
10	A
11	B
12	D
13	C
14	D
15	A
16	D
17	A
18	A
19	A
20	B
21	C
22	B
23	B
24	A
25	A
26	B
27	B

28	A
29	A
30	B
31	D
32	A
33	C
34	C
35	B
36	D
37	B
38	A
39	B
40	C
41	A
42	A
43	B
44	A
45	C
46	C
47	A
48	A
49	C
50	B
51	D
52	C
53	D
54	B
55	A
56	D
57	D
58	A

59	B
60	C
61	B
62	C
63	C
64	B
65	A
66	A
67	A
68	C
69	B
70	A
71	A
72	B
73	A
74	C

UNIT-IV

1. Which of the following defines a measure space?
   a. A set equipped with a topology
   b. A set equipped with a sigma-algebra and a measure
   c. A set equipped with a norm
   d. A set equipped with a metric

Hint: Consider the essential components of a measure space.

2. The Lebesgue measure of an interval [a, b] in ℝ is:
   a. (b - a)
   b. $(b - a)^2$
   c. b/a
   d. Undefined

Hint: What is the intuitive measure of an interval in the real line?

3. Which property characterizes a measure?
   a. Subadditivity
   b. Additivity
   c. Monotonicity
   d. All of the above

Hint: Think about the fundamental properties of measures.

4. The σ-algebra generated by a single set A in a set X is called:
   a. Power set of A
   b. σ-algebra of A
   c. Singleton set of A
   d. Trivial set of A

Hint: Consider the generated structure from a specific set.

5. The Lebesgue integral of a non-negative function f over a measurable set E is defined as:
   a. The supremum of the integral of simple functions
   b. The infimum of the integral of simple functions
   c. The limit of Riemann sums
   d. None of the above

Hint: How is the Lebesgue integral constructed for non-negative functions?

6. Which theorem states that if two functions are equal almost everywhere, they have the same Lebesgue integral over a measurable set?
   a. Radon-Nikodym theorem
   b. Fubini's theorem
   c. Lebesgue's dominated convergence theorem
   d. Egorov's theorem

Hint: Focus on the equality of functions over almost all points.

7. The Radon-Nikodym theorem deals with:
   a. Differentiation of measures
   b. Extension of measures
   c. Integral approximation
   d. Convergence of integrals

Hint: It involves the relationship between measures.

8. Which integral preserves the limit under certain conditions for a sequence of functions?

   a. Lebesgue integral
   b. Riemann integral
   c. Henstock-Kurzweil integral
   d. None of the above

Hint: Consider integrals that allow for more flexible limits.

9. The space of measurable functions is denoted as:

   a. L^2-space
   b. L^1-space
   c. L∞-space
   d. Lp-space

Hint: Think about the general space for measurable functions.

10. Which theorem deals with the interchange of integration for certain classes of functions?

   a. Lebesgue's differentiation theorem
   b. Fubini's theorem
   c. Egorov's theorem
   d. Vitali's convergence theorem

Hint: It involves the interchange of multiple integrals.

11. The space $L^2(R)$ consists of functions that are:

   a. Bounded
   b. Square-integrable
   c. Absolutely continuous
   d. Lipschitz continuous

Hint: Consider the property related to the square of the function.

12. The set of points where a function fails to be continuous is of measure:

a. Zero
b. Infinite
c. Non-measurable
d. Unpredictable

Hint: Consider the measure of the set of discontinuities.

13. Which function is always Lebesgue integrable on a finite measure space?

a. Indicator function of a measurable set
b. Discontinuous function
c. Unbounded function
d. All of the above

Hint: Think about the basic properties of integrability.

14. The dual space of L^1 consists of:

a. Radon measures
b. Lebesgue measures
c. Borel measures
d. Dirac measures

Hint: Consider the space that pairs with L^1 functions.

15. The property that states the measure of the union of disjoint sets is the sum of their individual measures is known as:

a. Countable additivity
b. Continuity from below
c. Countable subadditivity
d. Continuity from above

Hint: Think about the behavior of measures on unions.

16. Which theorem ensures the existence of Lebesgue measurable sets?

a. Carathéodory's criterion
b. Lebesgue's differentiation theorem
c. Hahn-Banach theorem
d. Tychonoff's theorem

Hint: It deals with the criteria for measurable sets.

17. The property that the measure of the empty set is zero is known as:

a. Null set property
b. Null measure property
c. Nullity property
d. Vacuous measure property

Hint: Think about the measure of the empty set.

18. Which theorem guarantees the existence of a sequence of simple functions converging to a measurable function pointwise?

a. Lusin's theorem
b. Beppo Levi's theorem
c. Radon-Nikodym theorem

d. Egorov's theorem

Hint: It involves a sequence approximating a function.

19. The absolute continuity of measures involves:
    a. Measures that are mutually singular
    b. Measures that are singular with respect to each other
    c. One measure being dominated by another
    d. Measures being comparable in all sets

Hint: Focus on the relationship between measures.

20. Which theorem asserts that for a sequence of measurable functions, convergence in measure implies convergence almost everywhere along a subsequence?
    a. Radon-Nikodym theorem
    b. Vitali's convergence theorem
    c. Lebesgue's dominated convergence theorem
    d. Egorov's theorem

Hint: It involves convergence properties.

21. The space $L\infty(X)$ consists of functions that are:
    a. Absolutely continuous
    b. Bounded almost everywhere
    c. Uniformly continuous
    d. Square-integrable

Hint: Consider the characteristic of the functions in this space.

22. A set is said to be measurable if it belongs to the:
    a. Sigma-algebra
    b. Topology
    c. Borel set
    d. Power set

Hint: Think about the criterion for a set to have a measure.

23. Which theorem ensures the existence of an integral representation for measures with respect to another measure?
    a. Lebesgue's differentiation theorem
    b. Radon-Nikodym theorem
    c. Fubini's theorem
    d. Egorov's theorem

Hint: It involves representation with respect to another measure.

24. The Lebesgue integral extends the Riemann integral by handling functions that are:
    a. Discontinuous
    b. Unbounded
    c. Oscillating
    d. All of the above

Hint: Consider the limitations of the Riemann integral.

25. The σ-finite property of a measure space implies:
    a. Countable additivity

b. Existence of a countable partition
c. Existence of a countable cover
d. All of the above

Hint: Think about the property related to the decomposition of the space.

26. Which theorem states that a function in L^1 is approximated by simple functions in the L^1 norm?
a. Lebesgue's dominated convergence theorem
b. Lebesgue's differentiation theorem
c. Lebesgue's convergence theorem
d. Lebesgue's approximation theorem

Hint: It involves the approximation of functions.

27. The measure of a countable set in a σ-finite measure space is:
a. Always zero
b. Always finite
c. Zero if the set is bounded
d. Zero if the set is disjoint

Hint: Consider the measure of countable sets in a σ-finite space.

28. Which theorem guarantees the existence of an outer measure corresponding to a given set function?
a. Carathéodory's criterion
b. Radon-Nikodym theorem
c. Lebesgue's differentiation theorem

d. Fubini's theorem

Hint: It involves the criteria for measures.

29. Which property characterizes a Lebesgue measurable set?

a. The boundary is measure-zero
b. The interior is empty
c. The complement is also measurable
d. All of the above

Hint: Think about the characteristics of measurable sets.

30. The space $L^1(X)$ consists of functions that are:

a. Square-integrable
b. Absolutely continuous
c. Integrable over the space
d. Bounded almost everywhere

Hint: Consider the nature of functions in this space.

31. Which property characterizes a function to be measurable?

a. Continuity of the function.
b. Differentiability of the function.
c. The pre-image of measurable sets is measurable.
d. The function is bounded.

Hint:A property of pre-images of sets under the function's domain.

32. Which theorem provides a condition for the Lebesgue integrability of a function based on another integrable function?

a. Monotone Convergence Theorem.
b. Lebesgue Dominated Convergence Theorem.
c. Fatou's Lemma.
d. Beppo Levi's Theorem.

Hint: It compares two functions and their absolute values

33.For which type of function is the Lebesgue integral always finite?

Hint: A specific characteristic concerning the function's behavior.

a. Functions that are bounded.
b. Continuous functions.
c. Functions with compact support.
d. Functions with countably many discontinuities.

Hint: A specific characteristic concerning the function's behaviour.

34.The integral $\int_0^1 \frac{1}{\sqrt{x}}\, dx$ is:

a. Finite.
b. Infinite.
c. Zero.
d. Oscillatory.

Hint: Consider the singularity in the integrand.

35. What property holds true for the integral of the sum of two integrable functions?

a. int (f+g) = int f + int g
b. int (f.g) = int f .int g
c. int (f-g) = int f - int g
d. int $(\frac{f}{g})$= {int f \int g}

Hint: Consider the linearity of integration.

36.Which theorem allows the interchange of limit and integral under certain conditions for a sequence of functions?

a. Monotone Convergence Theorem.
b. Dominate d Convergence Theorem.
c. Fatou's Lemma.
d. Beppo Levi's Theorem.

Hint: Concerns the convergence of functions.

37. The Lebesgue integral of the characteristic function of the interval [a, b] is:

a. a - b
b. b - a
c. 1
d. 0

Hint: Characteristic functions represent sets.

38. Which property characterizes a function that is Lebesgue integrable over a measurable set E?

a. The function is continuous on E.
b. The function is bounded on E.
c. The function is measurable on E.
d. The function is differentiable on E.

Hint: Focus on the function's behavior on the specified set.

39.For a function to be Lebesgue integrable, what property should hold for its integral over a set?

a. The integral is always finite.
b. The integral may be infinite.
c. The integral is zero.
d. The integral is positive.

Hint: Concerns the finiteness of the integral.

40. What is the Radon-Nikodym theorem concerned with?

a. Singular integrals
b. Measure theory
c. Differentiation of functions
d. Complex analysis

Hint: This theorem provides conditions for the existence of a certain type of function.

40. In the context of the Radon-Nikodym theorem, what does the term "absolute continuity" refer to?

a. A measure dominated by another measure
b. A measure equivalent to another measure
c. A measure concentrated on a null set
d. A measure concentrated on a countable set

Hint: It denotes one measure being dominated by another in a particular sense.

41. Which of the following is a necessary condition for the Radon-Nikodym theorem to hold between two measures?

a. Sigma-finiteness

b. Countable additivity
c. Continuity
d. Finite additivity

Hint: It ensures that the measures involved satisfy certain properties regarding their decomposition.

42. What is the Radon-Nikodym derivative used for?

a. Finding antiderivatives of functions
b. Differentiating vector-valued functions
c. Describing the relationship between measures
d. Evaluating improper integrals

Hint: It expresses how one measure changes with respect to another measure.

43. If μ and ν are measures such that ν is absolutely continuous with respect to μ, which of the following holds true according to the Radon-Nikodym theorem?

a. μ is absolutely continuous with respect to ν
b. ν is singular with respect to μ
c. There exists a measurable function representing their relationship
d. μ and ν are mutually singular measures

Hint: The theorem provides a representation of one measure with respect to another.

44. What does the Radon-Nikodym theorem establish regarding measures?

a. It establishes conditions for absolute continuity of measures.
b. It focuses on countable additivity properties of measures.
c. It defines the concept of singular measures.
d. It proves the existence of Lebesgue integrals for bounded functions.

Hint: It deals with a relationship between measures regarding absolute continuity.

45. Which property characterizes a signed measure that is absolutely continuous with respect to another measure?

a. Singularity
b. Discreteness
c. Continuity
d. Existence of a density function

Hint: Absolute continuity implies one measure is dominated by another in a specific way.

46. What does the Radon-Nikodym theorem provide for measures satisfying certain conditions?

a. An explicit formula for the derivative measure
b. A decomposition into singular and continuous parts
c. A method to compute Lebesgue integrals
d. A characterization of sigma-finite measures
    a. Hint: It's about expressing one measure in terms of another.

47. In terms of measures, what does it mean for one measure to be singular with respect to another?

   a. They share no common elements in their support.
   b. They are mutually absolutely continuous.
   c. One measure dominates the other.
   d. They have the same Radon-Nikodym derivative.

Hint: Singular measures are unrelated in terms of absolute continuity.

.

48. For what type of measures does the Radon-Nikodym theorem hold?

   a. Only for finite measures
   b. Only for sigma-finite measures
   c. Only for atomic measures
   d. For sigma-finite and absolutely continuous measures

Hint: It's about the conditions necessary for the theorem to apply.

49. A measure on a sigma-algebra \(\mathcal{F} \) is a function that satisfies:

a) Countable additivity
b) Finite additivity
c) Continuity
d) Differentiability

Hint: Consider the fundamental properties that define a measure.

50. The Lebesgue measure is an example of:
    a. A finite measure
    b. A countably additive measure
    c. A signed measure
    d. A finite additivity measure

Hint: Recall the properties and type of measure provided by the Lebesgue measure.

51. A signed measure assigns:
    a. Only non-negative values to sets
    b. Positive values to sets
    c. Both positive and negative values to sets
    d. Values from a discrete set to sets

Hint: Consider the nature of values assigned by a signed measure.

52. A measure that assigns both positive and negative infinity to sets is called:
    a. A finite measure
    b. An unsigned measure
    c. A complex measure
    d. A signed measure

Hint: Think about measures that can assign values beyond the real number line.

53. Which property characterizes a signed measure?
    a. Countable additivity
    b. Finite additivity

c. Monotonicity
d. Subadditivity

Hint: Consider the specific properties that define a signed measure.

54. The concept of a signed measure extends the notion of a measure by allowing:
    a. Only positive values for sets
    b. Negative values for sets
    c. Only finite values for sets
    d. Infinite values for sets

Hint: Consider how signed measures expand the scope of measures.

55. A measure that assigns values to sets with the property that $\mu(\emptyset) = 0$ is called:
    a. A probability measure
    b. A countably additive measure
    c. An unsigned measure
    d. A signed measure

Hint: Consider the special property concerning the null set for certain measures.

56. The outer measure of a set E is defined as:
    a. The supremum of the measures of open sets containing E
    b. The infimum of the measures of open sets containing E

c. The sum of the measures of open sets containing E
d. The product of the measures of open sets containing E

Hint: Think about the definition and computation of outer measures.

57. A measure space consists of:
a. A sigma-algebra and a measure defined on it
b. A measure and a sigma-algebra defined on it
c. A topology and a measure defined on it
d. An inner product space and a measure defined on it

Hint: Recall the components that make up a measure space.

58. Which of the following statements is true about the null set concerning measures?
a. Every null set is measurable
b. Every measurable set is a null set
c. The complement of a null set is also null
d. Null sets have finite measure

Hint: Consider the characteristics of null sets in measure theory.

59. A measure that assigns the value $+\infty$ to every non-empty set is termed as:
a. A finite measure
b. A sigma-finite measure
c. An unsigned measure
d. An atomic measure

Hint: Consider measures that assign infinite values to sets.

60. The concept of sigma-finiteness for a measure space involves:

a. The measure of the entire space being finite
b. The space being countably generated by sets of finite measure
c. The space consisting of countably many sets
d. The measure being finite on the sigma-algebra

Hint: Consider the properties associated with sigma-finite measure spaces.

61. Which property characterizes a sigma-finite measure space?

a. Every set has finite measure
b. The measure of the entire space is infinite
c. The measure space can be decomposed into countably many sets of finite measure
d. The measure space contains countably infinite sets

Hint: Think about the specific property that defines sigma-finite measure spaces.

62. A measure that assigns values to sets with the property that \(\mu(\emptyset) = 0 \) is called:

a. A sigma-finite measure
b. An unsigned measure
c. A complex measure
d. An atomless measure

Hint: Consider the special property concerning the null set for certain measures.

63. Which of the following is true about the Radon-Nikodym theorem?

a. It relates signed measures to unsigned measures
b. It characterizes the total variation of a measure
c. It characterizes the absolute continuity of measures
d. It characterizes the singularity of measures

Hint: Consider the theorem that addresses the relationship between measures.

64. A measure space consists of:

a. A measure and a topology
b. A sigma-algebra and a measure defined on it
c. An inner product space and a measure defined on it
d. A measure and a function defined on it

Hint: Consider the fundamental components that make up a measure space.

65. The Radon-Nikodym theorem establishes a relationship between:

a. Bounded functions
b. Signed measures and absolute continuity
c. Lebesgue integrals and Riemann integrals
d. Countable sets and uncountable sets

Hint: Consider the theorem that addresses a specific relationship between measures.

66. The Radon-Nikodym theorem is concerned with:
    a. The convergence of functions
    b. The decomposition of measures
    c. The integrability of functions
    d. The continuity of functions

Hint: Think about the specific aspect of measures addressed by the Radon-Nikodym theorem.

67. The Radon-Nikodym theorem is applied to:
    a. Signed measures
    b. Countably additive measures
    c. Finite measures
    d. Lebesgue measures

Hint: Consider the type of measures involved in the theorem.

68. The Radon-Nikodym theorem provides a way to decompose:
    a. Lebesgue integrals
    b. Riemann integrals
    c. Measures
    d. Functions

Hint: Consider the specific aspect that the theorem allows for decomposition.

69. The Radon-Nikodym theorem establishes a relationship between two measures, one being:
    a. Continuous
    b. Absolutely continuous with respect to the other
    c. Singular with respect to the other

d. Discontinuous

Hint: Consider the types of relationships between measures addressed by the theorem.

70. The Radon-Nikodym theorem is particularly useful in:

a. Calculating Lebesgue integrals
b. Decomposing complex functions
c. Defining inner products in functional analysis
d. Analysing the relationship between measures

Hint: Think about the application domain of the Radon-Nikodym theorem.

71. The Radon-Nikodym theorem is related to the concept of:

a. Outer measures
b. Countable sets
c. Measure decompositions
d. Continuity of functions

Hint: Consider the theorem's connection to specific concepts in measure theory.

72. The Radon-Nikodym theorem is a fundamental result in:

a. Riemann integration
b. Lebesgue integration
c. Fourier analysis
d. Functional analysis

Hint: Think about the field of study where the Radon-Nikodym theorem holds significance.

73. The Radon-Nikodym theorem allows for the representation of a measure in terms of:

a. Lebesgue integrals
b. Riemann integrals
c. A density function
d. A complex function

Hint: Consider the representation facilitated by the Radon-Nikodym theorem.

74. The Radon-Nikodym theorem characterizes the relationship between two measures in terms of:

a. Continuity
b. Absolute continuity
c. Countable additivity
d. Boundedness

Hint: Think about the specific type of relationship between measures addressed by the theorem.

75. The Radon-Nikodym theorem is often applied in:

a. Probability theory
b. Geometric analysis
c. Differential equations
d. Algebraic geometry

Hint: Consider the field of study where the Radon-Nikodym theorem finds applications.

76. The Radon-Nikodym theorem allows for the representation of a measure as:

a. A Lebesgue integral
b. A sum of measures
c. The product of measures
d. The derivative of a measure with respect to another

Hint: Think about the representation facilitated by the Radon-Nikodym theorem.

77. The Radon-Nikodym theorem is fundamental in understanding the concept of:

a. Integrability of functions
b. Absolute continuity of measures
c. Uniform convergence
d. Topological spaces

Hint: Consider the concept addressed by the Radon-Nikodym theorem in measure theory.

78. The Radon-Nikodym theorem allows for the comparison of measures through their:

a. Convergence properties
b. Density functions
c. Sigma-algebras
d. Outer measures

Hint: Consider the comparison facilitated by the Radon-Nikodym theorem between measures.

ANSWERS

S.NO	OPTIONS
1	B
2	A
3	D
4	B
5	A
6	C
7	A
8	A
9	D
10	B
11	B
12	A
13	A
14	A
15	A
16	A
17	B
18	D
19	C
20	B
21	B
22	A
23	B
24	D
25	D
26	D
27	A
28	A

29	D
30	C
31	C
32	B
33	A
34	B
35	A
36	B
37	C
38	A
39	B
40	B
41	A
42	C
43	C
44	A
45	D
46	B
47	A
48	D
49	A
50	B
51	C
52	C
53	A
54	B
55	C
56	A
57	B
58	C
59	C

60	B
61	C
62	B
63	C
64	B
65	B
66	B
67	A
68	C
69	B
70	D
71	C
72	B
73	C
74	B
75	A
76	D
77	B
78	B

UNIT-V

1. Which of the following is a fundamental property of a measure?

   a. Countable additivity
   b. Finite additivity
   c. Continuity
   d. Absolute continuity

   Hint: Consider the property that defines how a measure behaves on countable unions.

2. What does the outer measure generalize?

   a. Lebesgue measure
   b. Borel measure
   c. Counting measure
   d. Inner measure

   Hint: Think about the broader concept that outer measure encompasses.

3. Which of the following sets is NOT necessarily measurable with respect to the Lebesgue outer measure?

   a. Borel sets
   b. Vitali sets
   c. Cantor sets
   d. Jordan measurable sets

   Hint: Consider the properties of sets that make them measurable with respect to outer measures.

4. The Carathéodory extension theorem deals with the extension of which concept?

a. Inner measure
b. Outer measure
c. Lebesgue measure
d. Counting measure

Hint: Focus on the theorem that extends a certain measure to a larger class of sets.

5. Which property ensures that a set is measurable with respect to an outer measure?

a. Subadditivity
b. Countable additivity
c. Carathéodory criterion
d. Finite additivity

Hint: Think about the criterion that determines measurable sets based on the behavior of subsets.

6. What is the fundamental concept that outer measure extends?

a. Area and volume
b. Counting and cardinality
c. Inner measure
d. Lebesgue measure

Hint: Outer measure broadens a certain notion to a larger class of sets.

7. Which theorem provides a criterion for determining measurable sets using outer measures?

a. Carathéodory extension theorem
b. Vitali's covering theorem
c. Hahn-Banach theorem
d. Carathéodory criterion

Hint: This theorem specifies a condition for sets to be measurable based on outer measures.

8. The concept of measure extends which property of the outer measure?

a. Countable additivity
b. Subadditivity
c. Finite additivity
d. Continuity

Hint: Measures possess a specific property that outer measures do not.

Answer: A) Countable additivity

9. Which of the following sets may not necessarily be measurable with respect to outer measure?

a. Lebesgue measurable sets
b. Borel sets
c. Cantor sets
d. Vitali sets

Hint: Consider the properties of sets that are measurable with respect to outer measures.

10. What property characterizes measurable sets with respect to outer measures?

a. They satisfy countable additivity.
b. They form a sigma-algebra.

c. They have finite measure.
d. They are closed under countable unions and complements.

Hint: Think about the defining characteristics of sets measurable by outer measures.

11. What theorem deals with extending measures from smaller classes of sets to larger ones?

a. Carathéodory extension theorem
b. Lebesgue differentiation theorem
c. Fubini's theorem
d. Vitali's theorem

Hint: This theorem is concerned with extending measures to a broader class of sets.

12. Which of the following defines the concept of outer measure?

a. A function defined on a sigma-algebra of sets that satisfies countable additivity.
b. A function defined on a set that assigns non-negative values to subsets and is countably subadditive.
c. A function defined on a set that is countably additive.
d. A function defined on a sigma-algebra of sets that assigns non-negative values and is countably subadditive.

Hint: Outer measure extends a certain concept to a larger class of sets.

.

13. What is the primary aim of the Carathéodory extension theorem?

a. To extend measures from an algebra to a sigma-algebra.
b. To define Lebesgue integration for unbounded functions.
c. To establish the countable additivity property for measures.
d. To extend measures from an open set to a closed set.

Hint: Focus on the theorem that extends a measure's domain.

14. Which criterion specifies the conditions for a set to be measurable with respect to outer measures?

a. Carathéodory criterion
b. Hahn-Banach criterion
c. Cantor-Dedekind criterion
d. Lebesgue's criterion

Hint: This criterion provides the necessary condition for measurability.

15. The concept of a measure emphasizes which property that outer measure lacks?

a. Countable additivity
b. Subadditivity
c. Superadditivity
d. Finite additivity

Hint: Measures have a property that sets them apart from outer measures.

16. What property characterizes measurable sets with respect to outer measures?

a. They satisfy countable additivity.

b. They form a sigma-algebra.
c. They have finite measure.
d. They are closed under countable unions and complements.

Hint: Consider the defining characteristic of sets measurable by outer measures.

17. Which of the following is a property of outer measure?
a. Additivity
b. Countable subadditivity
c. Countable additivity
d. Finite additivity

Hint: Consider the type of additivity that outer measures typically exhibit.

18. The Carathéodory criterion helps in determining:
a. Measure of sets in an algebra
b. Measure of bounded sets
c. Measurability of sets based on outer measures
d. Measure of sets in a sigma-algebra

Hint: Focus on the criterion that establishes measurability using outer measures.

19. Which property characterizes sets measurable with respect to outer measures?
a. Countable additivity
b. Finite measure
c. Closure under countable unions and complements

d. Uniqueness under unions and intersections

Hint: Think about the defining characteristic of sets measurable by outer measures.

20. The purpose of defining outer measure is to:
   a. Measure only finite sets
   b. Measure only measurable sets
   c. Measure a larger class of sets than those measurable by a given measure
   d. Measure sets in Euclidean space

Hint: Consider the extension of measurement to a broader class of sets.

21. The Carathéodory extension theorem deals with the extension of:
   a. Outer measure to inner measure
   b. Measure to outer measure
   c. Measure from an algebra to a sigma-algebra
   d. Outer measure from closed sets to open sets

Hint: Focus on the extension of measures to a broader class of sets.

22. The Carathéodory Extension Theorem deals with the extension of:
   a. Linear functions to non-linear functions
   b. Measures from a smaller class of sets to a larger class
   c. Continuous functions to discontinuous functions
   d. Integrals from bounded intervals to unbounded intervals

Hint: Focus on the extension of measures from one set of sets to a broader set.

23. The primary objective of the Carathéodory Extension Theorem is to:
   a. Extend Riemann integrals to Lebesgue integrals
   b. Extend measures from an algebra to a sigma-algebra
   c. Extend functions from differentiability to integrability
   d. Extend limits of sequences to limits of series

Hint: Think about the theorem's purpose in relation to measures.

24. The Carathéodory Extension Theorem ensures the extension of measures from:
   a. Closed sets to open sets
   b. Disjoint sets to overlapping sets
   c. A ring of sets to a sigma-algebra
   d. Unbounded intervals to bounded intervals

Hint: Focus on the theorem that deals with the extension of measures.

25. The Carathéodory Extension Theorem provides conditions for extending measures that satisfy:
   a. Finite additivity
   b. Countable additivity
   c. Uncountable additivity
   d. Disjoint additivity

Hint: Think about the property necessary for the extension of measures.

26. The Carathéodory Extension Theorem allows the extension of measures from a smaller class of sets to a larger class that is:

a. Closed under finite unions and complements
b. Closed under countable unions and complements
c. Closed under countable intersections
d. Closed under finite intersections

Hint: Consider the property required for the extension of measures.

27. The Extension theorem focuses on:

a. Shrinking measures in larger spaces
b. Reducing measures from larger to smaller spaces
c. Extending measures from smaller to larger spaces
d. Comparing measures in different spaces

Hint: Think about the process of expanding measures to encompass broader spaces.

28. The product measure of two measure spaces (X, A,μ),(Y,B,ϑ) is defined on the space:

a. (XxY, AxB, μxϑ)
b. (X∪ Y, A ∪B, μ+ϑ)
c. (X∩ Y, A ∩B, $\mu \cap \vartheta$)
d. (X/Y, A/B, μ/ϑ)

Hint: Consider how measures are constructed on the product space.

29. The Extension theorem ensures the existence of measures that are consistent with the original measure on the smaller space while maintaining:

a. Finiteness
b. Additivity
c. Uniformity
d. Disjointness

Hint:Think about the properties of measures that remain intact during the extension process.

30. What is the Extension theorem in measure theory primarily used for?

a. Extending outer measures to measures
b. Defining inner measures
c. Calculating Lebesgue integrals
d. Establishing measurable functions

Hint: The Extension theorem deals with extending certain functions from a smaller to a larger domain in measure theory.

31. Which of the following statements is true regarding outer measure?

a. It is always finite
b. It is countably additive
c. It is subadditive
d. It satisfies the principle of exclusion

Hint: Consider the properties and definitions of outer measure, particularly its relation to sets.

32. In measure theory, a set is measurable if:

a. Its outer measure is zero

b. It is countable
c. It satisfies Carathéodory's criterion
d. Its inner measure is finite

Hint: Consider the conditions that define measurability in measure theory.

33. The Extension theorem provides an extension from outer measures to measures, allowing for a measure defined on a smaller sigma-algebra to be extended to a larger one. Which theorem is closely related to this concept?

a. Monotone convergence theorem
b. Fatou's lemma
c. Hahn-Kolmogorov theorem
d. Vitali covering theorem

Hint: Think about theorems that relate to extending functions or measures to larger spaces.

34. Which property characterizes the notion of a measurable set according to Carathéodory's criterion?

a. Inner regularity
b. Outer regularity
c. Approximation by open sets
d. Closure under countable unions and complements

Hint: Carathéodory's criterion is concerned with defining measurable sets based on specific properties.

35. The concept of outer measure is used to define the:

a. Lebesgue integral

b. Lebesgue measure
c. Riemann integral
d. Borel measure

Hint: Consider the measure-theoretic concepts that relate to outer measures.

36. Which of the following functions can be extended using the Extension theorem?

a. Any continuous function
b. Functions defined on a measurable space
c. Outer measures
d. Measures defined on a sigma-algebra

Hint: Focus on the purpose and scope of the Extension theorem.

37. If a set is measurable, then its complement is:

a. Always measurable
b. Not necessarily measurable
c. Measurable only in finite measure spaces
d. Measurable if it has finite outer measure

Hint: Consider the relationship between measurability and set operations.

38. The Extension theorem is essential in the construction of:

a. Lebesgue-Stieltjes measures
b. Haar measures
c. Lesbesgue integrals
d. Borel sets

Hint: Think about the specific measures or integrals where the Extension theorem is applied.

39. Which property distinguishes outer measure from a measure?

   a. Countable additivity
   b. Subadditivity
   c. Monotonicity
   d. Finite additivity

Hint: Focus on the fundamental differences between outer measure and measure.

40. Carathéodory's criterion is primarily concerned with determining:

   a. The convergence of series
   b. The existence of limits
   c. The measurability of sets
   d. The continuity of functions

Hint: Consider the specific aspect of sets that Carathéodory's criterion addresses.

41. Which theorem ensures the existence of measures that extend outer measures and satisfy specific properties on a larger space?

   a. Carathéodory's extension theorem
   b. Hahn-Kolmogorov theorem
   c. Radon-Nikodym theorem
   d. Lebesgue's dominated convergence theorem

Hint: Think about theorems related to extending measures from outer measures.

42. A set is said to be measurable if it satisfies which condition?
    a. The set is finite
    b. The set is bounded
    c. The set can be approximated from the inside and outside by open sets
    d. The set has a finite Lebesgue measure

Hint: Consider the criteria for measurability concerning open sets.

43. Which property characterizes the notion of a measurable set concerning its approximation by other sets?
    a. Inner regularity
    b. Outer regularity
    c. Closure under countable unions and complements
    d. Monotonicity

Hint: Think about the regularity properties and approximation of sets in measure theory.

44. The Extension theorem is crucial in measure theory because it allows for the extension of:
    a. Measures to outer measures
    b. Measures to larger sigma-algebras
    c. Outer measures to measures
    d. Lebesgue integrals to Riemann integrals

Hint: Consider the specific extension enabled by the Extension theorem.

45. The concept of outer measure extends the idea of measure by considering:

a. Finite sets
b. Countable sets
c. Uncountable sets
d. Compact sets

Hint: Consider the extension beyond the scope of traditional measures.

46. The outer measure of a set is defined as:

a. The infimum of the measures of open sets containing it
b. The supremum of the measures of open sets containing it
c. The sum of the measures of open sets containing it
d. The product of the measures of open sets containing it

Hint: Think about the definition and computation of outer measures.

47. A set is measurable if and only if:

a. Its outer measure is zero
b. Its outer measure is finite
c. Its outer measure equals its inner measure
d. Its outer measure is countable

Hint: Consider the criteria that define a measurable set.

48. The Extension theorem in measure theory deals with the extension of:

a. Outer measures to all sets
b. Inner measures to all sets

c. Finite measures to uncountable sets
d. Countably additive measures to all sets

Hint: Consider the specific extension addressed by the Extension theorem.

49. The Extension theorem states that every:
    a. Outer measure is a measure
    b. Measure is an outer measure
    c. Outer measure can be extended to a measure on a sigma-algebra
    d. Measure can be extended to an outer measure

Hint: Consider the direction of extension discussed in the Extension theorem.

50. The Extension theorem is concerned with extending:
    a. Finite measures to all sets
    b. Countably additive measures to all sets
    c. Outer measures to all sets
    d. Lebesgue measures to all sets

Hint: Think about the types of measures addressed by the Extension theorem.

51. A measure that is countably additive and defined on a sigma-algebra is called:
    a. An outer measure
    b. A finite measure
    c. A Lebesgue measure
    d. A signed measure

Hint: Consider the properties and scope of various measures.

52. The concept of measurability in measure theory is related to the:

a. Density of sets
b. Approximation of sets
c. Continuity of functions
d. Integrability of functions

Hint: Consider the defining characteristics of measurable sets in measure theory.

53. The Extension theorem is crucial in establishing the existence of measures that satisfy:

a. Finite additivity
b. Countable additivity
c. Subadditivity
d. Superadditivity

Hint: Consider the specific properties guaranteed by the Extension theorem.

54. Outer measures provide a way to:

a. Measure the inner content of sets
b. Measure all subsets of a space
c. Measure the boundary of sets
d. Measure uncountable sets only

Hint: Think about the purpose and scope of outer measures.

55. A set is measurable if its outer measure satisfies the:
   a. Carathéodory criterion
   b. Borel-Cantelli lemma
   c. Fatou's lemma
   d. Heine-Borel theorem

Hint: Recall the criterion that defines a measurable set in terms of outer measure.

56. The Extension theorem is fundamental in ensuring the existence of:
   a. Measures satisfying countable additivity
   b. Finite measures on uncountable sets
   c. Lebesgue measures on countable sets
   d. Outer measures on finite sets

Hint: Consider the role of the extension theorem in guaranteeing the existence of specific measures.

57. The Extension theorem establishes a connection between:
   a. Finite measures and outer measures
   b. Outer measures and Lebesgue measures
   c. Measures and integrals
   d. Inner measures and outer measures

Hint: Think about the relationship addressed by the Extension theorem.

58. Outer measures are essential in measure theory as they provide a way to:
   a. Compute integrals

b. Approximate measure spaces
c. Define topology on measure spaces
d. Approximate measures of non-measurable sets

Hint: Consider the utility of outer measures in measure theory.

59. The Extension theorem ensures that:

a. Every measure can be extended to an outer measure
b. Every outer measure can be extended to a measure
c. Every inner measure is an outer measure
d. Every outer measure is countably additive

Hint: Think about the direction of extension discussed in the Extension theorem.

ANSWERS

S.NO	OPTIONS
1	A
2	A
3	B
4	A
5	C
6	C
7	D
8	A
9	D
10	B
11	A
12	D
13	A
14	A
15	A
16	B
17	B
18	C
19	C
20	C
21	C
22	B
23	B
24	C
25	B
26	B
27	C
28	A

29	B
30	A
31	C
32	C
33	C
34	D
35	B
36	C
37	B
38	A
39	A
40	C
41	A
42	C
43	A
44	C
45	C
46	B
47	C
48	B
49	C
50	C
51	C
52	B
53	B
54	B
55	A
56	A
57	D
58	D
59	B

ABOUT THE AUTHOR

Mrs. M. Meenakshi was born in 1984 at Coimbatore. Completed her school in Vellakovil and College in Erode. Did UG & PG in Maharaja College for Women, Perundurai; M.Phil in Sri Vasavi College in Cithode and currently doing Ph.D in Sree GVG Vishalakshi College in Udumalpet. Worked as an Assistant Professor at Tirupur Kumaran College For Women at Tirupur from 2009 till 2017. From 2017 to till date working as an Assistant Professor in the Department of Mathematics at St.Joseph's College of Arts and Science for Women, Hosur. Published 10 National & International Conference Proceedings. Area of interest in research is Graph Theory.

.

9 789334 010725

Mrs. M. Meenakshi