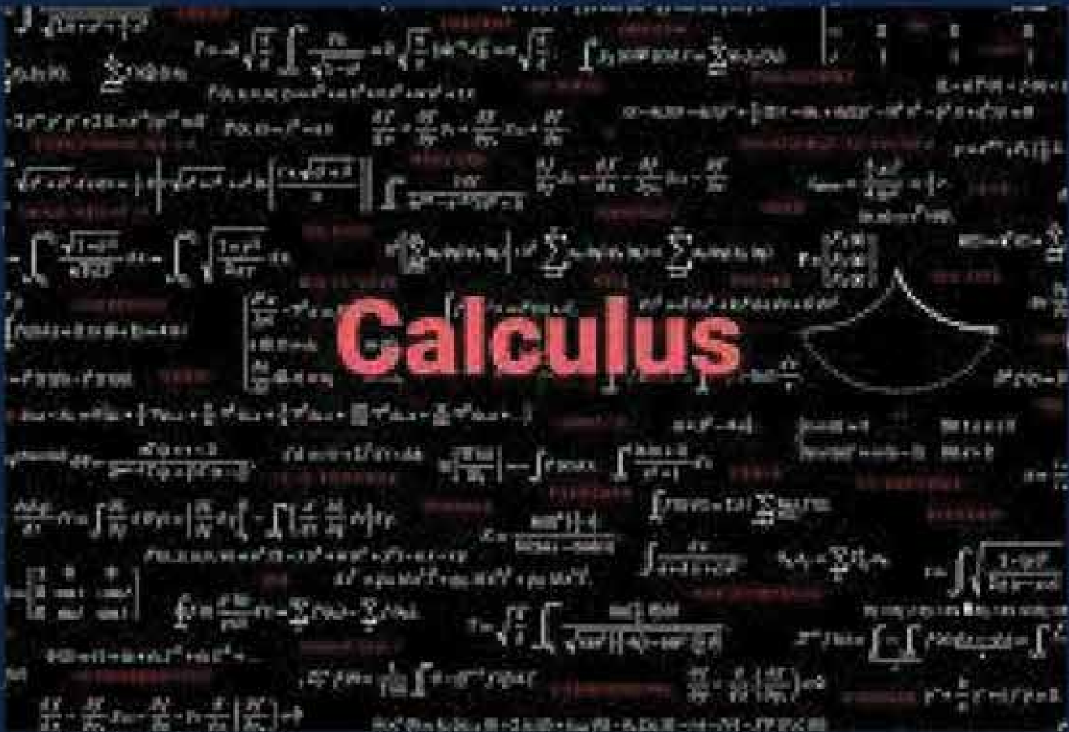

Solved MCQ on Allied Mathematics

St. Joseph's College of Arts and Science for Women, Hosur

MCQ ON ALLIED MATHEMATICS-I: ALGEBRA AND CALCULUS

UNIT – I :

Theory of Equations : Imaginary roots – Irrational roots – Formation of equations – Solutions of equations – Diminishing the roots of an equation & solutions – Removal of the second term of an equation & solutions – Descarte's rule of sign – Problems only. (Chapter6: Sections 4,9,10 & 11)

UNIT – II:

Matrices: Definition of Characteristic equation of a matrix –Characteristic roots of a matrix - Eigen values and the Corresponding Eigen vectors of matrix – Cayley Hamilton theorem (Statement only) – Verifications of Cayley Hamilton Theorem – Problems only. (Chapter 5)

UNIT – III :

Radius of Curvature: Formula of Radius of Curvature in Cartesian coordinates, Parametric coordinates and Polar coordinates (no proof for formulae) – Problems only. (Chapter11)

UNIT – IV :

Partial Differential Equations Formation of Partial Differential Equations by eliminating the arbitrary constant and arbitrary functions – Lagrange's Linear Partial Differential Equations – Problems only. (Chapter26)

UNIT – V :

Integration: Definite Integral : Simple properties of definite Integrals(Chap -15) – Bernoulli's Formula – Integration by parts – Simple problems ; Reduction formula for ∫ $sinnx\ dx\ \pi\ 2\ 0$, ∫ $cosnx\ dx\ \pi\ 2\ 0$, ∫ $e\ -x\ dx\ \infty\ 0$, ∫ $x\ ne\ ax\ dx$ simple problems. (Chapter16)

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1.THEORY OF EQUTION:

Every equation of the nth degree has a total 'n' real or imaginary roots. If α is the root of Equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$, i.e., $(x - \alpha)$ is the factor of the given polynomial $f(x)$.

In algebra, the study of algebraic equations, which are equations defined by a polynomial, is called the **theory of equations**. A polynomial is an expression consisting of one or more terms. The main difficulty of the theory of equations was to know when an algebraic equation has an algebraic solution. In this article, we will learn about the theory of equations and examples of solving equations.

The following are some important concepts covered under the theory of equations.

- Linear equations
- Simultaneous linear equations
- Finding the integer solutions of an equation or of a system of equations
- Systems of polynomial equations

Important Points to Remember

The important concepts in the theory of equations are given below:

1. The general form of a quadratic equation in x is given by $ax^2 + bx + c = 0$
2. The roots are given by $x = (-b\pm\sqrt{(b^2 - 4ac)})/2a$
3. If α and β are the roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$, then the sum of roots, $\alpha + \beta = -b/a$.

Product of roots, $\alpha\beta = c/a$

4. If the sum and product of roots are known, then the quadratic equation is given by $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

5. For a quadratic equation, $b^2 - 4ac$ is known as the discriminant denoted by D.
6. If $D = 0$, the equation will have two equal real roots.
7. If $D > 0$, then the equation will have two distinct real roots.
8. If $D < 0$, then the equation has no real roots.
9. The graph of a quadratic equation is a parabola. The parabola will open upwards if $a > 0$, and open downwards if $a < 0$.
10. If $a > 0$, when $x = -b/2a$, $f(x)$ attains its minimum value.
11. If $a < 0$, when $x = -b/2a$, $f(x)$ attains its maximum value.

Relationship between Roots and Coefficients

If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \ldots, \alpha_n$ are the roots of the quadratic equation:

$$a_0\text{-}x^n + a_1\text{-}x^{n-1} + a_2\text{-}x^{n-2} + a_3\text{-}x^{n-3} + a_4\text{-}x^{n-4} + \ldots\ldots\ldots\ldots\ldots + a_{n-1}x + a_n = 0$$

Then, the sum of roots:

$$\sum\alpha_1 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \ldots\ldots \alpha_n = -\frac{\mathbf{a_1}}{\mathbf{a_0}}$$

The sum of the product of roots taken two at a time:

$$\sum\alpha_1\alpha_2 = \frac{\mathbf{a_2}}{\mathbf{a_0}}$$

The sum of the product of roots taken three at a time:

$$\sum\alpha_1.\alpha_2.\alpha_3 = -\frac{\mathbf{a_3}}{\mathbf{a_0}}$$

Product of Roots:

$$\alpha_1.\alpha_2.\alpha_3.\alpha_4.\alpha_5 \ldots\ldots \alpha_n = (-1)^n \times \frac{a_n}{a_o}$$

Therefore, for a cubic equation

- $ax^3 + bx^2 + cx + d = 0$
- $\alpha 1 + \alpha_2 + \alpha_3 = -\frac{\mathbf{a_1}}{\mathbf{a_0}} = -\frac{\mathbf{b}}{\mathbf{a}}$
- $\alpha_1.\alpha_2 + \alpha_2.\alpha_3 + \alpha_3.\alpha_1 = \frac{\mathbf{a_2}}{\mathbf{a_0}} = \frac{\mathbf{c}}{\mathbf{a}}$
- $\alpha_1.\alpha_2.\alpha_3 = (-1)^n \times \frac{a_3}{a_o} = -\frac{d}{a}$

1.The transformed equation of x4+8x3+x−5=0 by eliminating second term is:

A

$x^4-24x^2+65x-55=0$

B

$x^4+24x^2-65x+55=0$

C

$x^4-24x^2-65x-55=0$

D

$x^4+24x^2+65x+55=0$

Solution

Here second term is nothing but x3

as we know it can be eliminated using transformation x=y+h

2. Factorise by splitting the middle term: $24x^2 - 65x + 21$

A

(3x- 7) (8x + 3)

B

(3x - 7) (8x - 3)

C

(3x + 7) (8x + 3)

D

(3x +7) (8x - 3)

Solution

The correct option is ***B***

(3x - 7) (8x - 3)

$$\begin{aligned}
&24x^2 - 65x + 21 \\
&\quad a = 24, \quad b = -65, \quad c = 21 \\
&\quad ac = 24 \times 21 = 56 \times 9 = (-56) \times (-9) \\
&\quad b = -65 = -56 + (-9) \\
&\quad 24x^2 - 65x + 21 \\
&= 24x^2 - 56x - 9x + 21 \\
&= 8x(3x - 7) - 3(3x - 7) \\
&= (8x - 3)(3x - 7)
\end{aligned}$$

3.Find the odd and even extensions of f(x) = x4−x3+x2 (x > 0)

[Assume the domain and range is x < 0 of the following function]

A

odd = $-x^4 - x^3 - x^2$, even = $x^4 + x^3 + x^2$

B

odd = $x^4 + x^3 + x^2$, even = $x^4 - x^3 + x^2$

C

odd = $x^4 - x^3 + x^2$, even = $-x^4 - x^3 - x^2$

D

odd = $-x^4 - x^3 - x^2$, even = $x^4 - x^3 + x^2$

Solution

The correct option is ***A***

odd = $-x^4 - x^3 - x^2$, even = $x^4 + x^3 + x^2$

Even extention of f(x): f(-x) = x4+x3+x2, x < 0

Odd extention of f(x): -f(-x) = −x4−x3−x2, x < 0

4.The degree of the polynomial obtained when 8−6x+x2−7x3+x5 is subtracted from x4−6x3+x2−3x+1 is:

A

$-x^5 + x^4 + x^3 + 3x$

B

$-x^5 + x^4 + x^3 - 7$

C

$-x^5 + x^4 + x^3 + 3x - 7$

D

$-x^5 + x^4 + 4x^3 + 3x - 7$

Solution

The correct option is **D.**

According to the question,

x4−6x3+x2−3x+1−(x5−7x3+x2−6x+8)

=x4−6x3+x2−3x+1−x5+7x3−x2+6x−8

=−x5+x4+x3+3x−7

Degree of the polynomial(that contains only one variable) is the highest exponent(power) of its various terms.

The terms of the polynomial obtained are x5,x4,x3,3x and 7.

Here, the term x5 has the highest power for the variable x and hence the degree of the given polynomial is 5.

5. x5+x4+x3+x2+x+1 by x3+1

Solution

x5+x4+x3+x2+x+1 by x3+1

$$x^3+1)\overline{x^5+x^4+x^3+x^2+x+1}\;(\;x^2+x+1$$

$$\underline{-x^5 \qquad\quad -x^2}$$

$$x^4+x^3+x+1$$

$$\underline{-x^4 \qquad -x}$$

$$x^3+1$$

$$\underline{-x^3 \quad -1}$$

$$\times$$

Quotient = x2+x+1

Remainder = 0

6. ∫(x+x2+x3+x4) dx is equal to

A

1+x+x2+x3+C

B

1+2x+3x2+4x3+C

C

x2+x3+x4+x5+C

D

x22+x33+x44+x55+C

Solution

The correct option is

***D** x22+x33+x44+x55+C*

∫(x+x2+x3+x4)dx

=∫x dx+∫x2dx+∫x3 dx+∫x4dx

=x1+11+1+x2+12+1+x3+13+1+x4+14+1

=x22+x33+x44+x55+C

7. A zero of $x^3 + 64$ is

A. 0

B. 4

C. 4i

D. -4

Answer : Option D

Explaination / Solution:

SOLUTION

$$x^3 + 64 = 0 \Rightarrow x^3 = -64 \Rightarrow x^3 = (-4)^3$$

$$x = -4 \text{ is the zero of } x^3 + 64$$

8. If *f* and *g* are polynomials of degrees *m* and *n* respectively, and if $h(x) = (f \square g)(x)$, then the degree of *h* is

A. mn

B. $m + n$

C. m^n

D. n^m

Answer : Option A

Explaination / Solution:

SOLUTION

f is a polynomial of degree m say $f(x) = a_m x^m$

g is a polynomial of degree n say $g(x) = b_n x^n$

$$f o g(x) = f(g(x)) = f(b_n x^n)$$

$$= a_m (b_n x^n)^m$$

$$= a_m b_n^m x^{mn}$$

$$\deg(f o g(x)) = mn$$

9. A polynomial equation in x of degree n always has

A. n distinct roots

B. n real roots

C. n imaginary roots

D. at most one root.

Answer : Option C
Explaination / Solution:

A polynomial of degree n always has n roots

10. If α , β , and γ are the zeros of x^3+px^2+qx+r , then $\sum\frac{1}{\alpha}$ is

A. $-q/r$

B. $-p/r$

C. q/r

D. $-q/p$

Answer : Option B
Explaination / Solution:

SOLUTION

Given α , β , γ are the roots of the equation

$$x^3 + px^2 + qx + r = 0$$

$$\alpha + \beta + \gamma = \frac{-p}{1} = -p$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{q}{1} = q$$

$$\alpha\beta\gamma = \frac{-r}{1} = -r$$

$$\therefore \quad \sum\frac{1}{\alpha} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

$$= \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = \frac{q}{-r}$$

$$\sum\frac{1}{\alpha} = \frac{-q}{r}$$

11. According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

A. -1

- **B.** 5/4
- **C.** 4/5
- **D.** 5

Answer : Option B
Explaination / Solution:

SOLUTION

The given polynomial is

$$P(x) = 4x^7 + 2x^4 - 10x^3 - 5$$

$$a_n = 4 \quad , \quad a_0 = -5$$

If $\frac{p}{q}$ is a zero of $P(x)$ where $(p, q) = 1$,

then p divides 5 and q divides 4.

p divides 5 $\Rightarrow$ The only numbers dividing 5 are

± 1 and ± 5

q dividies 4 $\Rightarrow$ The only numbers dividing 4 are

± 1, ± 2, ± 4.

$\therefore$ The possible $\frac{p}{q}$ are $\pm \frac{1}{1}$, $\pm \frac{5}{2}$, $\pm \frac{5}{4}$

$$P\left(\frac{5}{4}\right) = 4 \times \left(\frac{5}{4}\right)^7 + 2\left(\frac{5}{4}\right)^4 - 10\left(\frac{5}{4}\right)^3 - 5 \neq 0$$

$\therefore$ The rational number $\frac{5}{4}$ cannot be a rational zero of $P(x)$

12. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

- **A.** $|k| \leq 6$
- **B.** $k = 0$

C. $|k| > 6$

D. $|k| \geq 6$

Answer : Option D
Explaination / Solution:

SOLUTION

The equation of the given polynomial is

$$x^3 - kx^2 + 9x = 0$$

$$x(x^2 - kx + 9) = 0$$

$$x = 0 \quad \text{or} \quad x^2 - kx + 9 = 0$$

$x = 0$ is one root which is real. The other two roots are obtained by solving the equation $x^2 - kx + 9 = 0$

Given that the roots are real.

$\therefore$ The discriminant $b^2 - 4ac \geq 0$

$$(-k)^2 - 4 \times 1 \times 9 \geq 0$$

$$k^2 - 36 \geq 0$$

$$k^2 \geq 36$$

$$|k| \geq 6$$

$\therefore$ $x^3 - kx^2 + 9x = 0$ has all the three roots real if $|k| \geq 6$

13. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

A. 2

B. 4

C. 1

D. ∞

Answer : Option A

Explaination / Solution:

SOLUTION

The given equation is $\sin^4 x - 2\sin^2 x + 1 = 0$

$$(\sin^2 x)^2 - 2\sin^2 x + 1 = 0$$

Put $y = \sin^2 x$, then $y^2 - 2y + 1 = 0$

$$(y-1)^2 = 0$$

$$y = 1 \quad \text{or} \quad y = 1$$

$\sin^2 x = 1 \Rightarrow \sin x = \pm 1$

$x = \frac{\pi}{2} \in [0, 2\pi]$ satisfies $\sin x = \sin\frac{\pi}{2} = 1$

$x = \left(\pi + \frac{\pi}{2}\right) \in [0, 2\pi]$, satisfies

$$\sin x = \sin\left(\pi + \frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$$

$\therefore \frac{\pi}{2}, \left(\pi + \frac{\pi}{2}\right)$ are the two real numbers in $[0, 2\pi]$ satisfying $\sin x = \pm 1$

That is satisfying the given equation

$$\sin^4 x - 2\sin^2 x + 1 = 0$$

$\therefore$ Required number of real numbers $= 2$

14. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

A. $a \geq 0$

B. $a > 0$

C. $a < 0$

D. $a \leq 0$

Answer : Option C

Explaination / Solution:

SOLUTION

The equation of the given polynomial is

$$x^3 + 12x^2 + 10ax + 1999 = 0$$

Let $P(x) = x^3 + 12x^2 + 10ax + 1999$

If $a < 0$, the number of change of signs in $P(x) = 2$

$\therefore$ $P(x)$ cannot have more than 2 positive roots.

$\therefore$ Number of positive roots of $P(x)$ is $p = 1$ or 2

Total number of roots of $P(x)$ is $n = 3$

When $p = 1$, we have $n - p = 3 - 1 = 2$

which is even

$\therefore$ Number of positive roots of $P(x)$ is equal to 1

When $p = 2$, we have $n - p = 3 - 2 = 1$

which is not even

$\therefore$ Number of positive roots of $P(x)$ cannot be equal to 2.

Hence for the given polynomial has exactly one positive real root if $a < 0$

15. The polynomial $x^3 + 2x + 3$ has

A. one negative and two imaginary zeros

B. one positive and two imaginary zeros

C. three real zeros

D. no zeros

Answer : Option A

Explaination / Solution:

SOLUTION

$p(x) = x^3 + 2x + 3$, $p(x)$ has no change of sign

$\therefore$ $p(x)$ has no positive root.

$p(-x) = (-x)^3 + 2(-x) + 3$

$p(-x) = -x^3 - 2x + 3$

$p(-x)$ has only one sign change

$\therefore$ $p(x)$ has atmost one negative root

Thus $p(x)$ has no positive root and atmost one negative root.

Since $p(x)$ is a polynomial of degree 3 it has 3 roots.

$\therefore$ The remaining two roots must be imaginary roots.

Thus $p(x)$ has one negative roots and two imaginary roots.

16. The number of positive zeros of the polynomial is $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$

A. 0

B. n

C. $< n$

D. r

Answer : Option B

Explaination / Solution:

SOLUTION

Let $P(x) = \sum_{j=0}^{n} nC_r (-1)^r x^r$

The number of change in sign in $P(x)$ ' = n

$\therefore$ Number of positive zeros = n

17. Question

Factorise :7 – 12x – 4x^2 by splitting the middle term

Solution

7-14x+2x-4x^2

=(1-2x)(7+2x)

x=1/2 and -7/2

18. Question

Factorise by splitting the middle term $2x^2 - 11x + 12$

A

(2x-3)(x-4)

B

(x-3)(2x-4)

C

(x+3)(2x-4)

D

(2x+3)(x+4)

Solution

*The correct option is **A***

(2x-3)(x-4)

(2x-3)(x-4)

$2x^2-11x+12$

$=2x^2-8x-3x+12$

$=2x(x-4)-3(x-4)$

$=(2x-3)(x-4)$

19. $2x^2 - 11x + 15$ when factorised gives ________.

A

(2x + 5)(x + 6)

B

(x + 5)(x + 6)

C

(x - 5)(x - 6)

D

(x - 3)(2x - 5)

Solution

The correct option is ***D***

(x - 3)(2x - 5)

Splitting the middle term of the given expression such that the sum is -11 and product is 30, we get

$2x^2 - 11x + 15$

$\Rightarrow 2x^2 - 5x - 6x + 15$

$\Rightarrow x(2x-5) - 3(2x-5)$

$\Rightarrow (x-3)(2x-5)$

Hence, (x - 3) and (2x - 5) are the factors of $2x^2 - 11x + 15$.

20. Factorise: x2+10x+25

A

(x + 5)(x - 5)
B

$(x+5)^2$
C

$(x-5)^2$
D

(x + 5)(x + 2)

Solution

*The correct option is **B***
$(x+5)^2$

The expression x2+10x+25 can be written as,

=x2+2×5×x+52...(i)

Comparing (i) with the identity (a+b)2=a2+2ab+b2, we get

a2=x2⇒a=x

b2=52⇒b=5

⇒ x2+10x+25=x2+2(x)(5)+52 = (x+5)2

Therefore, the factors of x2+10x+25 are (x+5) and (x+5).
Sure thing! Let's dive into some multiple-choice questions on the theory of equations. I'll provide the questions, answer choices, and hints if needed. Feel free to take your time and let me know when you're ready for the answers.

PART 2

1. **Question:** What is the degree of the equation 3x^2 - 2x + 1 = 0?

a) 1

b) 2

c) 3

d) 4

Hint: The degree of an equation is the highest power of the variable.

2. **Question:** If x = -2 is a root of the equation x^2 + 4x + k = 0, what is the value of k?

a) 0

b) 4

c) -4

d) 8

Hint: Substitute x = -2 into the equation and solve for k.

3. **Question:** How many solutions does the equation 2x - 5 = 0 have?

a) 0

b) 1

c) 2

d) Infinite

Hint: Think about the nature of the solutions for a linear equation.

4. **Question:** What is the sum of the roots of the equation x^2 - 6x + 9 = 0?

a) 3

b) 6

c) 9

d) 0

Hint: Use Vieta's formulas to relate the coefficients to the sum of roots.

5. **Question:** If (x - 2) is a factor of the quadratic equation 2x^2 - 5x - 3 = 0, what is the other factor?

a) 2x + 3

b) x + 1

c) 2x - 3

d) x - 1

Hint: Use synthetic division or long division to find the other factor.

Certainly! Here are 30 multiple-choice questions (MCQs) on the theory of equations along with answers and hints:

Questions:

1. **What is the degree of the equation \(3x^2 - 2x + 1 = 0\)?**

- A) 1

- B) 2

- C) 3

- D) 0

2. **Which of the following is a quadratic equation?**

- A) \(2x + 1 = 0\)
- B) \(4x^2 - 6x + 3 = 0\)
- C) \(5x^3 + 2x^2 - 1 = 0\)
- D) \(x^2 + 3 = 0\)

3. **If \(x = 2\) is a root of the equation \(3x - k = 0\), what is the value of \(k\)?**

- A) 5
- B) 6
- C) 3
- D) 1

4. **What is the sum of the roots of the equation \(2x^2 - 5x + 3 = 0\)?**

- A) 2
- B) \(\frac{5}{2}\)
- C) \(\frac{3}{2}\)
- D) 5

5. **If \(x = -3\) is a root of \(2x^2 + px - 6 = 0\), what is the value of \(p\)?**

- A) -9
- B) 6
- C) 9
- D) -6

6. **What is the nature of the roots of the equation \(x^2 - 4x + 4 = 0\)?**

- A) Real and equal
- B) Real and distinct
- C) Imaginary
- D) Complex conjugates

7. **If $x + 2$ is a factor of $x^2 - 4x - 8$, what is the other factor?**

- A) $x + 2$
- B) $x - 2$
- C) $x + 4$
- D) $x - 4$

8. **The roots of $x^2 - 9 = 0$ are:**

- A) 3 and -3
- B) 9 and -9
- C) 3 and 9
- D) -3 and -9

9. **Which of the following is not a quadratic equation?**

- A) $3x^2 - 5x + 1 = 0$
- B) $4x - 1 = 0$
- C) $x^3 + 2x^2 - x + 1 = 0$
- D) $2x^2 + 7x = 0$

10. **If $x = 1$ is a solution to $2x^2 - 3x - k = 0$, what is the value of k?**

- A) -5
- B) -2

- C) 5

- D) 2

11. **What is the product of the roots of $4x^2 - 6x + 2 = 0$?**

- A) 2

- B) $\frac{1}{2}$

- C) 1

- D) $\frac{3}{2}$

12. **If $2x - 3$ is a factor of $4x^2 - 5x - 6$, what is the other factor?**

- A) $2x + 3$

- B) $4x + 3$

- C) $x - 2$

- D) $2x - 6$

13. **The roots of the equation $x^2 + 7x + 10 = 0$ are:**

- A) -2 and -5

- B) -5 and -2

- C) 2 and 5

- D) -10 and -7

14. **If $x = -2$ is a root of $x^2 + 3x + k = 0$, what is the value of k?**

- A) -4

- B) -2

- C) 4

- D) 2

15. **What is the value of \(a\) if \(x = 1\) is a root of \(ax^2 - 2ax + a = 0\)?**

- A) 1
- B) 2
- C) 0
- D) -1

16. **If \(x = -1\) is a root of \(x^3 - 2x^2 + x + 2 = 0\), what is the value of \(x + 1\)?**

- A) 0
- B) 1
- C) -2
- D) 2

17. **What is the discriminant of the equation \(3x^2 - 4x + 1 = 0\)?**

- A) 16
- B) 4
- C) -4
- D) 0

18. **If \(x = 3\) is a root of \(2x^2 - 5x + k = 0\), what is the value of \(k\)?**

- A) 0
- B) 3
- C) -3
- D) 6

19. **What is the sum of the roots of the equation \(x^2 + 6x + 9 = 0\)?**

- A) 3
- B) -3
- C) -6
- D) -9

20. **If \(x = 2\) is a root of \(4x^2 - 8x + p = 0\), what is the value of \(p\)?**
  - A) 4
  - B) 8
  - C) 2
  - D) -8

21. **What is the value of \(a\) if \(x = -1\) is a root of \(x^2 + ax - 1 = 0\)?**
  - A) -1
  - B) 0
  - C) 1
  - D) -2

22. **If \(x = 2\) is a root of \(x^2 - 3x + k = 0\), what is the value of \(k\)?**
  - A) 4
  - B) 2
  - C) 1

Sure thing! Here are 30 multiple-choice questions on the theory of equations, along with their answers and hints:

1. **Question:** What is the degree of the equation 3x^2 - 2x + 5 = 0?

a) 2

b) 3

c) 1

d) 0

Answer: a) 2

Hint: The highest power of the variable in the equation determines its degree.

2. **Question:** Which of the following is a quadratic equation?

a) $2x + 3 = 0$

b) x^2 - 4x + 7 = 0

c) 5x^3 + 2x^2 = 0

d) $4x - 1 = 0$

Answer: b) x^2 - 4x + 7 = 0

Hint: Quadratic equations have a degree of 2.

3. **Question:** What is the solution to the equation $2x - 8 = 0$?

a) $x = 4$

b) $x = -4$

c) $x = 2$

d) $x = -2$

Answer: a) $x = 4$

Hint: Move all the terms to one side to isolate the variable.

4. **Question:** If a quadratic equation has no real roots, what is the discriminant (Δ) value?

a) $\Delta > 0$

b) $\Delta = 0$

c) $\Delta < 0$

d) Δ can be any value

Answer: c) $\Delta < 0$

Hint: The discriminant determines the nature of roots.

5. **Question:** What is the sum of the roots of the equation $x^2 - 6x + 9 = 0$?

a) 6

b) 9

c) 3

d) 0

Answer: c) 3

Hint: Use the relationship between coefficients and roots in a quadratic equation.

2. Introduction To Eigenvalues And Eigenvectors

A rectangular arrangement of numbers in the form of rows and columns is known as a matrix. In this article, we will discuss **Eigenvalues and Eigenvectors Problems and Solutions.**

Consider a square matrix n × n. If X is the non-trivial column vector solution of the matrix equation AX = λX, where λ is a scalar, then X is the eigenvector of matrix A, and the corresponding value of λ is the eigenvalue of matrix A.

Suppose the matrix equation is written as A X – λ X = 0. Let I be the n × n identity matrix.

If I X is substituted by X in the equation above, we obtain

A X – λ I X = 0.

The equation is rewritten as (A – λ I) X = 0.

The equation above consists of non-trivial solutions if and only if the determinant value of the matrix is 0. The characteristic equation of A is Det (A – λ I) = 0. 'A' being an n × n matrix, if (A – λ I) is expanded, (A – λ I) will be the characteristic polynomial of A because its degree is n.

Properties of Eigenvalues

Let A be a matrix with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$.

The following are the properties of eigenvalues.

(1) The trace of A, defined as the sum of its diagonal elements, is also the sum of all eigenvalues,

$$tr(A) = \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i = \lambda_1 + \lambda_2 + \cdots + \lambda_n.$$

(2) The determinant of A is the product of all its eigenvalues,

$$det(A) = \prod_{i=1}^{n} \lambda_i = \lambda_1 \lambda_2 \cdots \lambda_n.$$

(3) The eigenvalues of the k^{th} power of A, that is, the eigenvalues of A^k, for any positive

integer k, are

$$\lambda_1^k, \ldots, \lambda_n^k.$$

(4) The matrix A is invertible if and only if every eigenvalue is nonzero.

(5) If A is invertible, then the eigenvalues of A^{-1} are

$$\frac{1}{\lambda_1}, \ldots, \frac{1}{\lambda_n}$$

and each eigenvalue's geometric multiplicity coincide. The characteristic polynomial of the inverse is the reciprocal polynomial of the original, the eigenvalues share the same algebraic multiplicity.

(6) If A is equal to its conjugate transpose, or equivalently if A is Hermitian, then every eigenvalue is real. The same is true for any real symmetric matrix.

(7) If A is not only Hermitian but also positive-definite, positive-semidefinite, negative-definite, or negative-semidefinite, then every eigenvalue is positive, non-negative, negative, or non-positive, respectively.

(8) If A is unitary, every eigenvalue has absolute value $|\lambda_i| = 1$.

(9) If A is a n×n matrix and $\{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ are its eigenvalues, then the eigenvalues of the matrix I + A (where I is the identity matrix) are $\{\lambda_1 + 1, \lambda_2 + 1, \ldots, \lambda_k + 1\}$.

1. *Eigenvalues Definition*: What Does It Mean For A Scalar Λ To Be An Eigenvalue Of A Matrix A?

   Hint: Think About The Relationship Between A Matrix, A Scalar, And Vectors.

2. *Characteristic Polynomial*: How Is The Characteristic Polynomial Of A Matrix Related To Its Eigenvalues?

   Hint: Express The Determinant Of (A - Λi) Where I Is The Identity Matrix.

3. *Cayley-Hamilton Theorem*: State The Cayley-Hamilton Theorem.

   Hint: Consider The Characteristic Polynomial Of A Matrix And Its Relationship To The Zero Matrix.

4. *Matrix Exponentiation*: How Is Matrix Exponentiation Involved In The Proof Of The Cayley-Hamilton Theorem?

Hint: Examine How E^(Ta) Relates To The Characteristic Polynomial.

5. *Eigenvalue Multiplicity*: Define The Multiplicity Of An Eigenvalue And Its Significance.

Hint: Consider How Many Times An Eigenvalue Appears In The Characteristic Polynomial.

1. What is the determinant of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$?

   (a) 2
   (b) 4
   (c) 6
   (d) None of the above

Solution: (c). You can compute this by cofactor expansion or row reduction.

2. What is the determinant of the matrix $\begin{bmatrix} \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \end{bmatrix}$?

   (a) 0
   (b) λ
   (c) λ^3
   (d) None of the above

Solution: (a). There are two identical rows.

3. Suppose the determinant of a 2×2 matrix A is equal to 5. What is the determinant of $2A$?

   (a) 5
   (b) 10
   (c) 20
   (d) 25
   (e) There is insufficient information to answer the question.

Solution: (c). Two rows get multiplied by 2, so the determinant is multiplied by 2 twice.

4. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

(e) There is insufficient information to answer the question

Solution: (c). The eigenvector-eigenvalue equation is $Ax = 1x$, so $A^2x = AAx = Ax = 1x = x$. Continuing in this manner we find $A^{50}x = x$.

5.

The components of pure shear strain in a sheared material are given in the matrix form:

$$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Here, Trace (ε) = 0. Given, P = Trace (ε^8) and Q = Trace (ε^{11})

The numerical value of (P + Q) is _______. (in integer)

ANSWER: 32

Explanation-

- The eigenvector of a matrix A is a vector represented by a matrix X such that when X is multiplied with matrix A, then the direction of the resultant matrix remains the same as vector X.
- AX = λX
    - where A is any arbitrary matrix, λ are eigen values and X is an eigen vector corresponding to each eigen value.Here, we can see that AX is parallel to X.So X is an eigen vector.

Some important properties of eigenvalues are-

- **If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigenvalues of A, then $\lambda_1^k, \lambda_2^k, \ldots\ldots, \lambda_n^k$ are the eigenvalues of A^k**
- **Sum of Eigen Values = Trace of A (Sum of diagonal elements of A)**

Given data and Calculation-

Given matrix = $\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$A - \lambda I = \begin{pmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{pmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-1-\lambda) - 1 = 0$$

$$\Rightarrow \lambda^2 - 2 = 0$$

$$\Rightarrow \lambda^2 = 2$$

$$\Rightarrow \lambda = \pm\sqrt{2}$$

Eigenvalues of ε^8 are $\left(\sqrt{2}\right)^8$ and $\left(-\sqrt{2}\right)^8$.

Eigenvalues of ε^{11} are $\left(\sqrt{2}\right)^{11}$ and $\left(-\sqrt{2}\right)^{11}$

P = Trace (ε^8) = 16 + 16 = 32

Q = Trace (ε^{11}).= 0

P + Q = 32

Additional Information

- Eigenvalues of real symmetric matrices are real.
- Eigenvalues of real skew-symmetric matrices are either pure imaginary or zero.
- Eigenvalues of unitary and orthogonal matrices are of unit modulus $|\lambda| = 1$
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigenvalues of A, then $k\lambda_1, k\lambda_2 \ldots\ldots k\lambda n$ are eigenvalues of kA.
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigenvalues of A, then $1/\lambda_1, 1/\lambda_2 \ldots\ldots 1/\lambda_n$ are eigenvalues of A^{-1}
- Eigenvalues of A = Eigen Values of A^T (Transpose)
- Product of Eigen Values = $|A|$
- Maximum number of distinct eigenvalues of A = Size of A

6.

Consider the following matrix.

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

1. x = 4, y = 10
2. x = 5, y = 8
3. x = -3, y = 9
4. x = -4, y = 10

Answer (Detailed Solution Below)

Option 4 : x = -4, y = 10

Eigenvalues Question 11 Detailed Solution

Concept:

If A is any square matrix of order n, we can form the matrix [A - λI], where I is the n^{th} order unit matrix. The determinant of this matrix equated to zero i.e. |A - λI| = 0 is called the characteristic equation of A.

The roots of the characteristic equation are called Eigenvalues or latent roots or characteristic roots of matrix A.

Properties of Eigenvalues:

(1) If λ is an Eigenvalue of a matrix A, then λ^n will be an Eigenvalue of a matrix A^n.

(2) If λ is an Eigenvalue of a matrix A, then kλ will be an Eigenvalue of a matrix kA where k is a scalar.

(3) Sum of Eigenvalues is equal to the trace of that matrix.

(4) The product of Eigenvalues of a matrix A is equal to the determinant of that matrix A.

(5) If λ is an Eigenvalue of matrix A, then λ^2 will be an Eigenvalue of matrix A^2.

Calculation:

Given:

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

Sum of eigen values = Trace (A) = 2 + y

Product of eigen values = |A| = 2y – 3x

∴ 4 + 8 = 2 + y ---- (i)

4 × 8 = 2y – 3x ---- (ii)

∴ 2 + y = 12 ---- (iii)

2y – 3x = 32 ---- (iv)

∴ Solving i) and ii) we get x = -4 and y = 10

7.

Which one of the following statements is true for all real symmetric matrices?

1. All the eigenvalues are real.
2. All the eigenvalues are positive.
3. All the eigenvalues are distinct.
4. Sum of all the eigenvalues is zero.

answer : All the eigenvalues are real.

Type of matrix	Definition	Eigenvalues
Symmetric matrix	A matrix 'A' is said to be symmetric matrix if **A = A^T** i.e. the matrix should be equal to its transpose matrix.	All **Eigenvalues** symmetric matrix are **real**. Eigenvectors corresponding to distinct eigenvalues are orthogonal.
Skew-Symmetric matrix	A matrix 'A' is said to be skew-symmetric matrix if **A = -A^T**	**Eigenvalues** of a real skew-symmetric matrix are **zero or purely imaginary**
Hermitian matrix	A Hermitian matrix (or self-adjoint matrix) is a complex square matrix that is equal to its conjugate transpose i.e. $A = (\bar{A})^T$	All **Eigenvalues** Hermitian matrix are **real** Eigenvectors corresponding to distinct eigenvalues are orthogonal
Skew-Hermitian matrix	A square matrix is said to be skew-Hermitian if it is equal to the negation of its complex conjugate transpose i.e. $A = -(\bar{A})^T$	**Eigenvalues** of a skew-Hermitian matrix are **zero or purely imaginary**
Orthogonal matrix	A square matrix with real numbers or elements is said to be an orthogonal matrix, if its transpose is equal to its inverse matrix i.e. **A^T = A^{-1} or AA^T = I**	Eigenvalues of the orthogonal matrix lie on the unit circle. They can be real or imaginary, but the magnitude is one. If the Eigenvalues are purely real, then they are ±1 If the Eigenvalues are purely imaginary, then they are ±j

8.

The eigen values of the matrix are

$$\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

1. $\pm \cos\alpha$

2. $\pm \sin\alpha$

3. $\tan\alpha$ & $\cot\alpha$

4. $\cos\alpha \pm \sin\alpha$

Option 4 : $\cos\alpha \pm \sin\alpha$

Eigenvalues Question 15 Detailed Solution

Concept:

The roots of the determinant of the matrix |SI - A| = 0 gives the eigenvalues.

Where I is an identity matrix and A is the given input matrix.

Properties of Eigenvalues:

1. The sum of Eigenvalues of a matrix A is equal to the trace of that matrix A

2. The product of Eigenvalues of a matrix A is equal to the determinant of that matrix A

Calculation:

Given:

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Eigen values are given by **|SI - A| = 0**

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} cos_\alpha & sin_\alpha \\ sin_\alpha & cos_\alpha \end{bmatrix} = 0$$

$$\begin{matrix} s - cos_\alpha & -sin_\alpha \\ -sin_\alpha & s - cos_\alpha \end{matrix} = 0$$

$(s - \cos\alpha)^2 - \sin^2\alpha = 0$

$(s - \cos\alpha) = \pm \sin\alpha$

$s = \cos\alpha \pm \sin\alpha$

Hence the solution is option (4)

9. FOR THE MATRIX A = $\begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, A^{-1} IS GIVEN BY

A.

$A^2 - 2A$

B.

$A^2 + 2A + 31$

C.

$A^2 — 2A — I$

D.

A - 31

10. Characteristic equation for the matrix A=[1234] is

A

$A^2+5A-2I=O$

B

$A^2-5A+2I=O$

C

$A^2-5A-2I=O$

D

$A^2+5A+2I=O$

Solution

The correct option is C A2−5A−2I=O

For characteristics equation : f(λ)=|A−λI|

⇒f(λ)=|||1−λ234−λ|||⇒f(λ)=(1−λ)(4−λ)−6⇒f(λ)=λ2−5λ−2

∴f(A)=O⇒A2−5A−2I=O

11\. Let $A \in M_2(\mathbb{R})$.

Which of the following statements is/are true?

1. If $(tr(A))^2 > 4\ det(A)$, Then A is diagonalizable over $\mathbb{R}$.
2. If $(tr(A))^2 = 4\ det(A)$, Then A is diagonalizable over $\mathbb{R}$.
3. If $(tr(A))^2 < 4\ det(A)$, Then A is diagonalizable over $\mathbb{R}$.
4. All of these

Concept:

(i) A matrix A is diagonalizable if all its eigenvalues are real and distinct

(iii) Non-zero nilpotent matrix is not diagonalizable

(iii) Characteristic equation of a 2 × 2 matrix A is $x^2 - tr(A)x + det(A) = 0$

Explanation:

$A \in M_2(\mathbb{R})$

Option 1): Characteristic equation

$x^2 - tr(A)x + det(A) = 0$

If $(tr(A))^2 > 4\ det(A)$ then all the roots are real and distinct

so eigenvalues of A are real and distinct

Then A is diagonalizable over ℝ.

Option (1) is true

Option 2): Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Then tr(A) = 0 and det(A) = 0

So $(tr(A))^2 = 4\ det(A)$

But A is a non-zero nilpotent matrix so not diagonalizable.

Option (2) is false

Option 3): If $(tr(A))^2 < 4\ det(A)$

take A = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

We get the Eigen Values in Complex Number

then all the roots of $x^2 - tr(A)x + det(A) = 0$ are complex which does not belongs to ℝ

Then A is diagonalizable over ℝ.

Option (3) is false

Therefore, Correct Option is Option 1).

12.

What is the determinant of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$?

(a) 2
(b) 4
(c) 6
(d) None of the above

Solution: (c). You can compute this by cofactor expansion or row reduction.

13. *Question*: What is an eigenvalue of a matrix?

- A. A matrix with all elements equal
- B. A scalar related to the matrix
- C. The determinant of the matrix
- D. A matrix raised to a power

Hint: Eigenvalues are associated with scalars.

14. *Question*: How do you find eigenvalues of a matrix A?

- A. By multiplying all elements of A
- B. Solving $\det(A - \lambda I) = 0$
- C. Taking the trace of A
- D. Adding all elements of A

Hint: Consider the characteristic equation.

15. *Question*: What does the characteristic polynomial of a matrix represent?

- A. The sum of matrix elements
- B. Eigenvalues of the matrix
- C. Inverse of the matrix
- D. Trace of the matrix

Hint: It involves the determinant of $(A - \lambda I)$.

16. *Question*: How many eigenvalues can a square matrix have?

- A. Only one
- B. At most two

- C. Equal to the matrix size
- D. Infinite

Hint: Think about the dimensions of the matrix.

17. *Question*: What does the algebraic multiplicity of an eigenvalue represent?

- A. Number of times the eigenvalue appears in the matrix
- B. The degree of the characteristic polynomial
- C. The rank of the matrix
- D. The determinant of the matrix

Hint: It's related to the characteristic polynomial.

18. *Question*: What is the characteristic equation of a matrix A?

- A. $\det(A + \lambda I) = 0$
- B. $\det(A - \lambda I) = 0$
- C. $\text{trace}(A + \lambda I) = 0$
- D. $\text{trace}(A - \lambda I) = 0$

Hint: Think about the form of the equation for finding eigenvalues.

19. *Question*: If a matrix has distinct eigenvalues, what can be said about its eigenvectors?

- A. They are identical
- B. They are linearly independent
- C. They are scalar multiples of each other
- D. They are orthogonal

Hint: Consider the relationship between eigenvalues and eigenvectors.

20. *Question*: How are eigenvalues affected when you multiply a matrix by a scalar?

- A. They remain the same
- B. They are multiplied by the scalar
- C. They are divided by the scalar
- D. They are squared

Hint: Think about the properties of scalar multiplication.

21. *Question*: What is the geometric multiplicity of an eigenvalue?

- A. The number of times the eigenvalue appears in the matrix
- B. The number of linearly independent eigenvectors for that eigenvalue
- C. The sum of all eigenvalues
- D. The determinant of the matrix

Hint: It's related to the eigenvectors.

22. *Question*: In a 3x3 matrix, what is the maximum number of distinct eigenvalues it can have?

- A. 1
- B. 2
- C. 3
- D. 4

Hint: Consider the dimensions of the matrix.

23. *Question*: What does the Cayley-Hamilton theorem state?

- A. Every matrix has at least one eigenvalue
- B. A matrix is equal to its characteristic polynomial
- C. The sum of eigenvalues is zero
- D. The determinant of a matrix is zero

Hint: Think about the relationship between a matrix and its characteristic polynomial.

24. *Question*: How does the Cayley-Hamilton theorem relate to the characteristic polynomial of a matrix?

- A. They are unrelated
- B. The characteristic polynomial is a solution to the theorem
- C. The characteristic polynomial is a factor in the theorem
- D. The theorem contradicts the characteristic polynomial

Hint: Consider how the characteristic polynomial is involved in the theorem.

25. *Question*: What is the key expression used in proving the Cayley-Hamilton theorem for a matrix A?

- A. $\det(A + \lambda I) = 0$
- B. $\det(A - \lambda I) = 0$
- C. $\mathrm{trace}(A + \lambda I) = 0$
- D. $\mathrm{trace}(A - \lambda I) = 0$

Hint: It involves the determinant of $(A - \lambda I)$.

26. *Question*: If the characteristic polynomial of a matrix is $\lambda^2 - 5\lambda + 6$, what is the

Cayley-Hamilton theorem's implication for the matrix?

- A. A^2 - 5A + 6I = 0
- B. A^3 - 5A^2 + 6A = 0
- C. A^2 + 5A - 6I = 0
- D. A^3 + 5A^2 - 6A = 0

Hint: Replace λ with the matrix A in the characteristic polynomial.

27. *Question*: What role does the identity matrix play in the Cayley-Hamilton theorem?

- A. It is not involved in the theorem
- B. It is added to the matrix A
- C. It is subtracted from the matrix A
- D. It is multiplied by the scalar λ

Hint: Pay attention to the form of the matrices involved in the theorem.

Eigenvalues and Eigenvectors Solved Problems

Example 1: Find the eigenvalues and eigenvectors of the following matrix.

Solution:

$$A=\begin{pmatrix}2 & 2\\ 1 & 3\end{pmatrix}, f_A(x)=|xI-A|=(x-2)(x-3)-2=x^2-5x+4, \lambda_1,\lambda_2=[5\pm\sqrt{(25-16)}]/2=4,1.$$

$$[A-4I\,|\,0]=\left(\begin{array}{cc|c}-2 & 2 & 0\\ 1 & -1 & 0\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1 & -1 & 0\\ 0 & 0 & 0\end{array}\right)\Rightarrow v_1=\begin{pmatrix}1\\1\end{pmatrix}\text{ is an eigen - vector for e.v. }\lambda_1=4.$$

$$[A-I\,|\,0]=\left(\begin{array}{cc|c}1 & 2 & 0\\ 1 & 2 & 0\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1 & 2 & 0\\ 0 & 0 & 0\end{array}\right)\Rightarrow v_2=\begin{pmatrix}-2\\1\end{pmatrix}\text{ is an eigen - vector for e.v. }\lambda_2=1.$$

Solve:

Example 2: Find all eigenvalues and corresponding eigenvectors for the matrix A if

$$\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Solution:

Example 3: Consider the matrix

$$A = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 3 & a \\ 4 & -2 & -1 \end{bmatrix}$$

for some variable 'a'. Find all values of 'a', which will prove that A has eigenvalues 0, 3, and −3.

What do you mean by eigenvalues?

Eigenvalues are the special set of scalar values associated with the set of linear equations in the matrix equations.

Q2Can the eigenvalue be zero?

Yes, the eigenvalue can be zero.

Q3Can a singular matrix have eigenvalues?

Every singular matrix has a 0 eigenvalue.

Q4How to find the eigenvalues of a square matrix A?

Use the equation det(A-λI) = 0 and solve for λ. Determine all the possible values of λ, which are the required eigenvalues of matrix A.

RADIUS OF CURVATURE:

The radius of the approximate circle at a particular point is the radius of curvature. The curvature vector length is the radius of curvature. The radius changes as the curve moves. Denoted by R, the radius of curvature is found out by the following formula.

Formula for Radius of Curvature

$$R = \frac{(1 + (\frac{dy}{dx})^2)^{3/2}}{|\frac{d^2y}{dx}|}$$

In polar coordinates r=r(Θ), the radius of curvature is given by

$$\rho = \frac{1}{K}\frac{[r^2 + (\frac{dr}{d\theta})^2]^{3/2}}{|r^2 + 2(\frac{dr}{d\theta})^2 - r\frac{d^2r}{d\theta^2}|}$$

Solved Examples Using Curvature Radius Formula

1. **Question:** Find the radius of the curvature for $y = 5x^3 - x + 14$ at x=2.

Solution:

$y = 5x^3 - x + 14$

$dy/dx = 15x^2 - 1$

$d^2y/dx^2 = 30x$

$$R = \frac{(1 + (\frac{dy}{dx})^2)^{3/2}}{|\frac{d^2y}{dx}|}$$

$$= \frac{[1 + (15x^2 - 1)^2]^{\frac{3}{2}}}{30x}$$

$$= \frac{[1 + 225x^4 - 30x^2 + 1]^{\frac{3}{2}}}{30x}$$

$$= \frac{[225x^4 - 30x^2 + 2]^{\frac{3}{2}}}{30x}$$

$$R = \frac{(1 + (\frac{dy}{dx})^2)^{3/2}}{|\frac{d^2y}{dx}|}$$

$$= \frac{[1 + (15x^2 - 1)^2]^{\frac{3}{2}}}{30x}$$

$$= \frac{[1 + 225x^4 - 30x^2 + 1]^{\frac{3}{2}}}{30x}$$

$$= \frac{[225x^4 - 30x^2 + 2]^{\frac{3}{2}}}{30x}$$

1.If the curve is a straight line, what can be said about its radius of curvature?

a. It is undefined

b. It is infinite

c. It is constant

d. It is always negative

2. The radius of curvature at any point of the curve is the reciprocal of:

a. Tangent to the curve

b. Slope of the curve

c. Normal to the curve

d. Second derivative of the curve

3. For a circle, what is the radius of curvature at any point on the circumference?

a. Equal to the radius of the circle

b. Double the radius of the circle

c. Zero

d. Undefined

4. If the curve is a straight line, what can be said about its radius of curvature?

a. It is undefined

b. It is infinite

c. It is constant

d. It is always negative

- Hint: Consider the geometric interpretation of a straight line.

Answer: b

6. The radius of curvature at any point of the curve is the reciprocal of:

a. Tangent to the curve

b. Slope of the curve

c. Normal to the curve

d. Second derivative of the curve

- Hint: Think about the relationship between curvature and derivatives.

Answer: d

7. For a circle, what is the radius of curvature at any point on the circumference?

a. Equal to the radius of the circle

b. Double the radius of the circle

c. Zero

d. Undefined

- Hint: Consider the nature of a circle.

Answer: a

8. In the equation \(y = e^x\), what is the radius of curvature at any point?

a. \(e^{3/2}\)

b. \(e^{1/2}\)

c. \(e\)

d. \(e^2\)

- Hint: Use the formula for the radius of curvature.

Answer: c

9. The radius of curvature is minimum when the curve is:

a. Concave upward

b. Concave downward

c. A straight line

d. A circle

- Hint: Consider the relationship between curvature and the shape of the curve.

Answer: d

10. If the curve is given by \(y = x^2\), what is the radius of curvature at the point (1, 1)?

a. \(2\)

b. \(1\)

c. \(\frac{1}{2}\)

d. \(0\)

- Hint: Use the formula for the radius of curvature.

Answer: b

11. The radius of curvature for a circle is:

a. Constant

b. Variable

c. Zero

d. Infinite

- Hint: Consider the nature of a circle.

Answer: a

11. What is the relationship between the radius of curvature and the curvature of a curve?

a. Directly proportional

b. Inversely proportional

c. No relationship

d. Exponential relationship

- Hint: Think about how curvature is related to derivatives.

Answer: a

12. The radius of curvature is maximum when the curve is:

a. Concave upward

b. Concave downward

c. A straight line

d. A circle

- Hint: Consider the relationship between curvature and the shape of the curve.

Answer: d

13. For a curve given by $y = \sin x$, what is the radius of curvature at $x = \pi/4$?

a. $\frac{1}{\sqrt{2}}$

b. $\frac{\sqrt{2}}{2}$

c. 1

d. 2

- Hint: Use the formula for the radius of curvature.

Answer: b

14. In the equation $y = \ln x$, what is the radius of curvature at $x = 1$?

a. 1

b. 0

c. e

d. $\frac{1}{e}$

- Hint: Use the formula for the radius of curvature.

Answer: a

15. The radius of curvature at the origin for the curve $y = x^3$ is:

a. 0

b. 1

c. Undefined

d. 3

- Hint: Use the formula for the radius of curvature.

Answer: a

16. For the curve given by $y = e^{-x}$, what is the radius of curvature at any point?

a. e^{-1}

b. 1

c. e

d. e^2

- Hint: Use the formula for the radius of curvature.

Answer: a

17. If $y = \sqrt{x}$, what is the radius of curvature at the point (4, 2)?

a. $\frac{3}{2}$

b. 2

c. 3

d. 4

- Hint: Use the formula for the radius of curvature.

Answer: a

18. The radius of curvature of the curve $y = x^2 + 1$ at the point (1, 2) is:

a. 1

b. 2

c. 3

d. 4

- Hint: Use the formula for the radius of curvature.

Answer: b

19. The radius of curvature for a circle with radius \(R\) is:

a. \(R\)

b. \(2R\)

c. \(0\)

d. R/2

- Hint: Consider the nature of a circle.

Answer: a

20. If the curve is given by \(y = \cos x\), what is the radius of curvature at \(x = \pi/2\)?

a. \(1\)

b. \(0\)

c. 1/2)

d. 1/√‾2}}\)

- Hint: Use the formula for the radius of curvature.

Answer: a

21. The radius of curvature for a straight line is:

a. Constant

b. Variable

c. Zero

d. Infinite

- Hint: Consider the geometric interpretation of a straight line.

Answer: d

22. For the curve $y = x^3 - 3x$, what is the radius of curvature at the point (2, 2)?

a. 2

b. 4

c. 1

d. 3

- Hint: Use the formula for the radius of curvature.

23. What is the equation for the radius of curvature (R) in terms of a curve represented by $y = f(x)$?

a. $R = \frac{1}{{f'(x)}}$

b. $R = \frac{{(1 + [f'(x)]^2)^{3/2}}}{{f''(x)}}$

c. $R = \frac{{1}}{{f''(x)}}$

d. $R = \frac{{f''(x)}}{{(1 + [f'(x)]^2)^{3/2}}}$

Hint: Use the formula for the radius of curvature.

Answer: b. $R = \frac{{(1 + [f'(x)]^2)^{3/2}}}{{f''(x)}}$

24. If the curve is a straight line, what can be said about its radius of curvature?

a. It is undefined

b. It is infinite

c. It is constant

d. It is always negative

Hint: Consider the nature of a straight line.

Answer: b. It is infinite

25. The radius of curvature at any point of the curve is the reciprocal of:

a. Tangent to the curve

b. Slope of the curve

c. Normal to the curve

d. Second derivative of the curve

Hint: Relate the radius of curvature to a geometric property.

Answer: b. Slope of the curve

26. For a circle, what is the radius of curvature at any point on the circumference?

a. Equal to the radius of the circle

b. Double the radius of the circle

c. Zero

d. Undefined

Hint: Think about the specific characteristics of a circle.

Answer: a. Equal to the radius of the circle

27. In the equation \(y = e^x\), what is the radius of curvature at any point?

a. \(e^{3/2}\)

b. \(e^{1/2}\)

c. \(e\)

d. \(e^2\)

Hint: Apply the formula for the radius of curvature.

Answer: c. \(e\)

28. The radius of curvature is minimum when the curve is:

a. Concave upward

b. Concave downward

c. A straight line

d. A circle

Hint: Think about the geometric properties of concavity.

Answer: d. A circle

29. If the curve is given by \(y = x^2\), what is the radius of curvature at the point (1, 1)?

a. \(2\)

b. \(1\)

c. \(\frac{1}{2}\)

d. \(0\)

Hint: Use the formula for the radius of curvature at a specific point.

Answer: b. \(1\)

30. The radius of curvature for a circle is:

a. Constant

b. Variable

c. Zero

d. Infinite

Hint: Consider the nature of a circle.

Answer: a. Constant

31. What is the relationship between the radius of curvature and the curvature of a curve?

a. Directly proportional

b. Inversely proportional

c. No relationship

d. Exponential relationship

Hint: Relate the terms in the context of curvature.

Answer: a. Directly proportional

32. The radius of curvature is maximum when the curve is:

a. Concave upward

b. Concave downward

c. A straight line

d. A circle

Hint: Think about the geometric properties of concavity.

Answer: d. A circle

33. For a curve given by \(y = \sin x\), what is the radius of curvature at \(x = \pi/4\)?

a. \(\frac{1}{\sqrt{2}}\)

b. $\frac{\sqrt{2}}{2}$

c. 1

d. 2

Hint: Apply the formula for the radius of curvature at a specific point.

Answer: b. $\frac{\sqrt{2}}{2}$

34. In the equation $y = \ln x$, what is the radius of curvature at $x = 1$?

a. 1

b. 0

c. e

d. $\frac{1}{e}$

Hint: Use the formula for the radius of curvature at a specific point.

Answer: b. 0

35. The radius of curvature at the origin for the curve $y = x^3$ is:

a. 0

b. 1

c. Undefined

d. 3

Hint: Consider the characteristics of the given curve.

Answer: a. 0

36. For the curve given by $y = e^{-x}$, what is the radius of curvature at any point?

a. e^{-1}

b. \(1\)

c. \(e\)

d. \(e^2\)

Hint: Apply the formula for the radius of curvature.

Answer: a. \(e^{-1}\)

37. If \(y = \sqrt{x}\), what is the radius of curvature at the point (4, 2)?

a. \(\frac{3}{2}\)

b. \(2\)

c. \(3\)

d. \(4\)

Hint: Use the formula for the radius of curvature at a specific point.

Answer: c. \(3\)

38. The radius of curvature of the curve \(y = x^2 + 1\) at the point (1, 2) is:

a. \(1\)

b. \(2\)

c. \(3\)

d. \(4\)

Hint: Apply the formula for the radius of curvature at a specific point.

Answer: b. \(2\)

39. The radius of curvature for a circle with radius \(R\) is:

a. \(R\)

b. \(2R\)

c. \(0\)

d. \(\frac{R}{2}\)

Hint: Relate the radius of curvature to the properties of a circle.

Answer: a. \(R\)

39.. If the curve is given by \(y = \cos x\), what is the radius of curvature at \(x = \pi/2\)?

a. \(1\)

b. \(0\)

c. \(\frac{1}{2}\)

d. 1/2

Q.1: What is the order of differential equation y'' + 5y' + 6 = 0?

A. 0

B. 1

C. 2

D. 3

Answer: C. 2

Explanation: Given, differential equation y'' + 5y' + 6 = 0.

The highest order derivative present in the differential equation is y''. Hence, the order is 2.

2. What is the degree of differential equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$?

A. 2

B. 3

C. 4

D. 5

Answer: A. 2

Explanation: The degree is the power raised to the highest order derivative. Therefore, in the given differential equation, $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$, the degree will be power raised to y'''.

So, the answer is 2.

Q.3: Find the order of differential equations: $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$

A. 2

B. 1

C. 0

D. Undefined

Answer: A. 2

Answer: A. 2

Explanation: Given, the differential equation is:

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

Or we can write:

$2x^2 y'' - 3y' + y = 0$

Order is the highest derivative in the differential equation. Therefore, the order is 2.

(Q.4: Find the degree of the differential equation: $(1 + \frac{dy}{dx})^3 = (\frac{dy}{dx})^2$

A. 0

B. 1

C. 2

D. 3

Answer: C. 3

Explanation: Given, the differential equation is

: $(1+\frac{dy}{dx})^3 = (\frac{dy}{dx})^2$

We can expand it and get:

$$1+3\frac{dy}{dx}+3(\frac{dy}{dx})^2+(\frac{dy}{dx})^3=(\frac{dy}{dx})^2$$

The exponent of highest derivative is the degree. Therefore, the degree is 3.

Q.5: The number of arbitrary constants in the particular solution of a differential equation of third order is:

A. 3

B. 2

C. 1

D. 0

Answer: D. 0

Explanation: The solution free from arbitrary constants i.e., the solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation.

Q.6: What is the differential equation of the family of circles touching the y-axis at the origin?

A. $2xyy' + x^2 = y^2$

B. $2xyy'' + x' = y^2$

C. $2xyy' - x^2 = y^2$

D. $xyy' + x^2 = y^2$

Answer: A. $2xyy' + x^2 = y^2$

Explanation: Let the center of the circle touch the y- axis at origin lies on the x-axis.

Say, (k, 0) be the center of the circle.

Hence, it touches the y – axis at origin, its radius is p.

Now, the equation of the circle with center (p, 0) and radius (p) is

$\Rightarrow (x - k)^2 + y^2 = k^2$

$\Rightarrow x^2 + k^2 - 2xk + y^2 = p^2$

Shifting k and – 2xk to RHS then it becomes: k^2 and 2xk

$\Rightarrow x^2 + y^2 = k^2 - k^2 + 2kx$

$\Rightarrow x^2 + y^2 = 2kx$ ….(i)

Differentiating equation on both sides, we have,

$\Rightarrow 2x + 2yy' = 2k$

$\Rightarrow x + yy' = k$

Now, on substituting the value of 'k' in the equation (i), we get,

$\Rightarrow x^2 + y^2 = 2(x + yy')x$

$\Rightarrow 2xyy' + x^2 = y^2$

Therefore, $2xyy' + x^2 = y^2$ is the required differential equation.

Q.7: Solution of differential equation x.dy – y.dx = Q represents:

A. a rectangular hyperbola

B. parabola whose vertex is at the origin

C. straight line passing through the origin

D. a circle whose centre is at the origin

Answer: C. straight line passing through the origin

Q.9: Which of the following is a second-order differential equation?

A. $(y')^2 + x = y^2$

B. $y'y'' + y = \sin x$

C. $y''' + (y'')^2 + y = 0$

D. $y' = y^2$

Answer: B. $y'y'' + y = \sin x$

Explanation: The order of $y'y'' + y = \sin x$ is 2. Thus, it is a second-order differential equation.

Q.10: The curve for which the slope of the tangent at any point is equal to the ratio of the abscissa to the ordinate of the point is:

A. an ellipse

B. parabola

C. circle

D. hyperbola

Answer: D. hyperbola

UNIT: 4

1) If (d/dx) f(x) is g(x), then the antiderivative of g(x) is

a. f(x)
b. f'(x)
c. g'(x)
d. None of the above

Answer: **(a) f(x)**

Given:

(d/dx) f(x) = g(x)

We know that the integration is the inverse process of differentiation, then the antiderivative of g(x) is f(x).

Hence, option (a) f(x) is the correct answer.

2) $_0\int^2 x^2\, dx$ =

a. 2
b. ⅔
c. 8/3
d. None of these

Answer: **(c) 8/3**

Explanation:

$_0\int^2 x^2\, dx = [x^3/3]_0^2$

Now, apply the limits, we get

$_0\int^2 x^2\, dx = (2^3/3) - 0 = 8/3$

Hence, option (c) is the correct answer.

3) $_0\int^2 (x^2 + 3)dx$ equals

a. 24/3
b. 25/3
c. 26/3
d. None of the above.

Answer: **(c) 26/3**

$_0\int^2 (x^2 + 3)dx = [(x^3/3)+ 3x]_0^2$

$_0\int^2 (x^2 + 3)dx = [(2^3/3)+ 3(2)] – 0 = (8/3)+6 = (8+18)/3 = 26/3.$

Hence, option (c) 26/3 is the correct answer.

4) If ∫ 2^x dx = f(x) + C, then f(x) is

a. 2^x
b. $2^x \log_e 2$
c. $2^x / \log_e 2$
d. $2^{x+1}/x+1$

Answer: **(c) $2^x / \log_e 2$**

Explanation: We know that differentiation is the inverse process of integration.

Therefore, $(d/dx)(2^x / \log_e 2) = (1/\log_e 2).2^x. \log_e 2 = 2^x$.

Hence, option (c) is the correct answer.

5) $_1\int^2 dx/x^2$ equals

a. 1
b. -1
c. 2
d. ½

Answer: **(d) ½**

$_1\int^2 dx/x^2 = {}_1\int^2 x^{-2} dx = [x^{-1}/-1]_1^2$

Now, apply limits, we get

$_1\int^2 dx/x^2 = (2^{-1}/-1) – (1^{-1}/-1)$

$_1\int^2 dx/x^2 = (1/-2)+1 = (1-2)/-2= -1/-2 = ½$

Hence, $_1\int^2 dx/x^2 = ½$.

6) ∫$\cot^2 x$ dx equals to

a. cot x – x + C

b. $-\cot x - x + C$
c. $\cot x + x + C$
d. $-\cot x + x + C$

Answer: **(b) $-\cot x - x + C$**

Explanation:

We know that $\cot^2 x = \operatorname{cosec}^2 x - 1$

$\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1) \, dx = -\cot x - x + C$. [Since, $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$]

Hence, the correct answer is option (b) $-\cot x - x + C$.

7) $\int_0^{\pi} \sin^2 x \, dx =$

a. $\pi/2$
b. $\pi/4$
c. 2π
d. 4π

Answer: **(a) $\pi/2$**

Explanation: $\int_0^{\pi} \sin^2 x \, dx = (½) \int_0^{\pi} (1-\cos 2x) \, dx$

Now, integrate the function and apply the limits, we get

$\int_0^{\pi} \sin^2 x \, dx = (½)(\pi-0) = \pi/2$.

Hence, option (a) $\pi/2$ is the correct answer.

8) $\int_0^4 3x \, dx$ equals

a. 12
b. 24
c. 48
d. 86

Answer: **(b) 24**

Explanation:

$\int_0^4 3x \, dx = 3\int_0^4 x \, dx = 3[x^2/2]_0^4$

Now, apply the limits, we get

$_0\int^4 3x\,dx = 3[(4^2/2) - 0]$

$_0\int^4 3x\,dx = 3[8-0] = 24$

Hence, $_0\int^4 3x\,dx = 24$.

9) Integrate $_0\int^2 (x^2+x+1)\,dx$

a. 15/2
b. 20/5
c. 20/3
d. 3/20

Answer: **(c) 20/3**

Explanation:

$_0\int^2 (x^2+x+1)\,dx = [(x^3/3)+(x^2/2)+x]_0^2$

$_0\int^2 (x^2+x+1)\,dx = [(2^3/3) + (2^2/2) + 2] - 0$

$_0\int^2 (x^2+x+1)\,dx = (8/3) + 2+2 = (8/3)+4$

$_0\int^2 (x^2+x+1)\,dx = (8+12)/3 = 20/3.$

Hence, option (c) 20/3 is the correct answer.

10) If $\int \sec^2(7-4x)dx = a \tan(7-4x) + C$, then value of a is

a. -4
b. -¼
c. 3
d. 7

Answer: **(b) -¼**

Explanation:

Given: $\int \sec^2(7-4x)dx = a \tan(7-4x) + C$

$\int \sec^2(7-4x)dx = \{[\tan(7-4x)]/-4\} + C$

$\int \sec^2(7-4x)dx = (-¼) \tan(7-4x) + C$

Hence, the value of a is -¼.

11. ∫ 1/x3 dx is

(a) $\frac{-3}{x^2}+c$ (b) $\frac{-1}{2x^2}+c$

(c) $\frac{-1}{3x^2}+c$ (d) $\frac{-2}{x^2}+c$

Ans: (b)

Hint: $= \int x^{-3}dx = \frac{x^{-3+1}}{-3+1} = \frac{x^{-2}}{-2} = \frac{-1}{2x^2}+c$

12. ∫ 2xdx is

(a) 2x log 2 + c

(b) 2x + c

(c) $\frac{2^x}{\log 2}+c$

(d) $\frac{\log 2}{2^x}+c$

Ans: (c)

Hint: $= \frac{2^x}{\log 2}+c$

13 $\int \frac{\sin 2x}{2\sin x}dx$ is

(a) sin x + c

(b) 1/2 sin x + c

(c) cos x + c

(d) 1/2 cos x + c

14. $\int \frac{\sin 5x - \sin x}{\cos 3x} dx$ is

(a) − cos2x + c

(b) − cos2x + c

(c) – 1/4 cos 2x + c

(d) −4 cos 2x + c

Hint: $\int \frac{\sin 5x - \sin x}{\cos 3x} dx$

$$= \int \frac{2\cos\left(\frac{5x+x}{2}\right)\cdot \sin\left(\frac{5x-x}{2}\right)\cdot dx}{\cos 3x}$$

[since $\sin \alpha - \sin \beta = 2\cos\frac{(\alpha+\beta)}{2}\cdot \sin\frac{(\alpha-\beta)}{2}$]

$$= 2\int \frac{\cancel{\cos 3x}\,.\sin 2x}{\cancel{\cos 3x}} dx$$

$$= 2\int \sin 2x\, dx$$

$$= \cancel{2}\left(-\frac{\cos 2x}{\cancel{2}}\right) + c$$

$$= -\cos 2x + c$$

15. ∫ logx/x dx , x > 0 is

(a) 1/2 (log x) 2 + c

(b) – 1/2 (log x)2

(c) 2/x2 + c

(d) 2/x2 + c

Hint: $= \int \frac{\log x}{x} dx, x > 0$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore \quad I = \int t \cdot dt = \frac{t^2}{2} + c$

$= \frac{1}{2}(\log x)^2 + c$

16.

$\int \frac{e^x}{\sqrt{1+e^x}} dx$ is

(a) $\frac{e^x}{\sqrt{1+e^x}} + c$ (b) $2\sqrt{1+e^x} + c$

(c) $\sqrt{1+e^x} + c$ (d) $e^x\sqrt{1+e^x} + c$

Ans: (b)

Hint: $I = \int \frac{e^x dx}{\sqrt{1+e^x}}$

Put $1 + e^x = t \quad \Rightarrow e^x dx = dt$

$\therefore \quad I = \int \frac{dt}{\sqrt{t}} = \int t^{\frac{-1}{2}} dt = \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{t} + c$

$= 2\sqrt{1+e^x} + c$

17.

$\int \sqrt{e^x}\, dx$ is

(a) $\sqrt{e^x} + c$ (b) $2\sqrt{e^x} + c$

(c) $\frac{1}{2}\sqrt{e^x} + c$ (d) $\frac{1}{2\sqrt{e^x}} + c$

Ans: (b)

Hint: $\int \sqrt{e^x}\, dx = \int (e^x)^{\frac{1}{2}} dx = \int e^{\frac{x}{2}} dx$

$$= \frac{e^{\frac{x}{2}}}{\frac{1}{2}} + c$$

$$= 2e^{\frac{x}{2}} + c = 2e^{x \times \frac{1}{2}} + c$$

$$= 2\sqrt{e^x} + c$$

18. ∫ e2x[2x2 + 2x]dx

(a) e2xx2 + c

(b) xe2x + c

(c) 2x2e2 + c

(d) x2ex /2 + c

Hint: $\int e^{2x} [2x^2 + 2x] dx$

Let $f(x) = x^2 \Rightarrow f'(x) = 2x$

We know $\int e^{ax} (a \cdot f(x) + f'(x))\, dx = e^{ax} \cdot f(x) + c$

Here $a = 2$

$\int e^{2x} (2 \cdot f(x) + f'(x))\, dx = e^{2x} \cdot f(x) + c$

$$= e^{2x} \cdot x^2 + c$$

19. $\int \frac{e^x}{e^x+1} dx$ is

(a) log |ex / ex + 1| + c

(b) log |ex + 1/ ex | + c

(c) log |ex| + c

(d) log |ex + 1| + c

Hint: $I = \int \frac{e^x}{e^x+1} dx$

Put $t = e^x + 1 \Rightarrow dt = e^x \cdot dx$

$I = \int \frac{dt}{t} = \log |t| + c$

$= \log |e^x + 1| + c$

20. $\int \left[\frac{9}{x-3} - \frac{1}{x+1}\right] dx$ is

(a) log |x – 3| – log |x + 1| + c

(b) log |x – 3| + log |x + 1| + c

(c) 9 log |x – 3| – log |x + 1| + c

(d) 9 log |x – 3| + log |x + 1| + c

Hint: $I = \int \left[\frac{9}{x-3} - \frac{1}{x+1}\right] dx$

$= 9 \cdot \int \frac{1}{x-3} dx - \int \frac{dx}{x+1}$

$= 9 \cdot \log | x - 3| - \log |x + 1| \; c$

21.

$\int \frac{2x^3}{4+x^4}\, dx$ is

(a) $\log|4+x^4|+c$ (b) $\frac{1}{2}\log|4+x^4|+c$

(c) $\frac{1}{4}\log|4+x^4|+c$ (d) $\log\left|\frac{2x^3}{4+x^4}\right|+c$

Ans: (b)

Hint: $I = \int \frac{2x^3}{4+x^4}\, dx$

Put $t = 4 + x^4$

$\Rightarrow dt = 0 + 4x^3\, dx$

$\frac{dt}{2} = 2 \cdot x^3\, dx$

$I = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log|t|+c$

$= \frac{1}{2}\log|4+x^4|+c$

22.

$\int \frac{dx}{\sqrt{x^2-36}}$ is

(a) $\sqrt{x^2-36}+c$ (b) $\log\left|x+\sqrt{x^2-36}\right|+c$

(c) $\log\left|x-\sqrt{x^2-36}\right|+c$ (d) $\log\left|x^2+\sqrt{x^2-36}\right|+c$

Ans: (b)

Hint: $\int \frac{dx}{\sqrt{x^2-36}}$

$$= \int \frac{dx}{\sqrt{x^2-6^2}}$$

$$= \log\left|x+\sqrt{x^2-36}\right|+c$$

23.

$\int \frac{2x+3}{\sqrt{x^2+3x+2}}\,dx$ is

(a) $\sqrt{x^2+3x+2}\ +c$

(b) $2\sqrt{x^2+3x+2}+c$

(c) $\log\left(x^2+3x+2\right)+c$

(d) $\frac{2}{3}\left(x^2+3x+2\right)^{\frac{3}{2}}+c$

Ans: (b)

Hint: $I = \int \frac{(2x+3)}{\sqrt{x^2+3x+2}}\,dx$

Put $t = x^2+3x+2$

$$dt = (2x+3)dx$$

$$\therefore \quad I = \int \frac{dt}{\sqrt{t}} = \int t^{-\frac{1}{2}}\,dt$$

$$= \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c = \frac{t^{\frac{1}{2}}}{\frac{1}{2}}+c$$

$$= 2\sqrt{t}+c$$

$$= 2\sqrt{x^2+3x+2}f+c$$

24. $\int_0^1 (2x+1)\, dx$ is

(a) 1

(b) 2

(c) 3

(d) 4

Hint: $\int_0^1 (2x+1)dx = \left[2\frac{x^2}{2}+x\right]_0^1$

$= \left[x^2+x\right]_0^1$

$= (1+1)-(0+0)$

$= 2-0=2$

25. $\int_2^4 \frac{dx}{x}$ is

(a) log 4

(b) 0

(c) log 2

(d) log 8

Hint: $\int_2^4 \frac{dx}{x} = [\log x]_2^4 = \log 4 - \log 2$

$= \log\frac{4}{2} = \log 2$

26. $\int_0^{\infty} e^{-2x}\, dx$ is

(a) 0

(b) 1

(c) 2

(d) 1/2

Hint: $\int_0^{\infty} e^{-2x}\, dx = \left[\frac{e^{-2x}}{-2}\right]_0^{\infty}$

$= -\frac{1}{2}\left[e^{-\infty} - e^{0}\right]$

$= -\frac{1}{2}(0-1) = -\frac{1}{2}(-1)$

$= \frac{1}{2}$

27. $\int_{-1}^{1} x^3\, e^{x^4}\, dx$ is

(a) 1

(b) $2\int_0^1 x^3\, e^{x^4}\, dx$

(c) 0

(d) ex4

Hint: $\int_{-1}^{1} x^3 e^{x^4} dx$

Let $f(x) = x^3 e^{x^4}$

$f(-x) = (-x)^3 e^{(-x)^4}$

$= -x^3 e^{x^4} = -f(x)$

28. If $f(x)$ is a continuous function and a < c < b ,then

$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ is

(a) $\int_a^b f(x)\,dx - \int_a^c f(x)\,dx$

(b) $\int_a^c f(x)\,dx - \int_a^b f(x)\,dx$

(c) $\int_a^b f(x)\,dx$

(d) 0

Ans: (c)

29. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$ is

(a) 0

(b) 2

(c) 1

(d) 4

Hint: The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$

$$= 2\int_0^{\frac{\pi}{2}} \cos x\, dx$$

[$\because$ cos x is an even function]

$$= 2(\sin x)_0^{\frac{\pi}{2}}$$

$$= 2\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= 2\,(1-0) = 2$$

30. $\int_0^1 \sqrt{x^4(1-x)^2}\, dx$ is

(a) 1 /12

(b) −7 /12

(c) 7 /12

(d) −1 /12

Hint: $\int_0^1 \sqrt{x^4(1-x)^2}\, dx = \int_0^1 x^2(1-x)\,dx$

$$= \int_0^1 (x^2 - x^3)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \left(\frac{1}{3} - \frac{1}{4}\right) - 0$$

$$= \frac{4-3}{12} = \frac{1}{12}$$

31. If

If $\int_0^1 f(x)\,dx = 1$, $\int_0^1 x\, f(x)\,dx = a$ and

$\int_0^1 x^2\, f(x)\,dx = a^2$, then $\int_0^1 (a-x)^2\, f(x)\,dx$

is

(a) 4a2

(b) 0

(c) 2a2

(d) 1

Hint: If $\int_0^1 f(x)dx = 1$, $\int_0^1 x f(x) dx = a$,

$$\int_0^1 x^2 f(x) dx = a^2, \text{ then } \int_0^1 (a-x)^2 f(x) dx$$

$$= \int_0^1 \left(a^2 + x^2 - 2ax\right) f(x) dx$$

$$= \int_0^1 a^2 f(x) dx + \int_0^1 x^2 f(x) dx - \int_0^1 2ax\, f(x) dx$$

$$= a^2 \int_0^1 f(x) dx + a^2 - 2a \int_0^1 x\, f(x) dx$$

$$= a^2 (1) + a^2 - 2a(a)$$

$$= 2a^2 - 2a^2 = 0$$

32. The value of $\int_2^3 f(5-x)\,dx - \int_2^3 f(x)\,dx$ is

(a) 1

(b) 0

(c) −1

(d) 5

Hint: $\int_2^3 f(5-x)\,dx - \int_2^3 f(x)\,dx$

$= \int_2^3 f(5-x)\,dx - \int_2^3 f(2+3-x)\,dx$

$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$= \int_2^3 f(5-x)\,dx - \int_2^3 f(5-x)\,dx$

$= 0$

33. $\int_0^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ is

(a) 20/3

(b) 21/3

(c) 28/3

(d) 1/3

Hint: $\int_0^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$$= \int_0^4 \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^4$$

$$= \left[\frac{2}{3}\left(x^{\frac{3}{2}}\right) + 2\sqrt{x}\right]_0^4$$

$$= \left[\frac{2}{3}x\sqrt{x} + 2\sqrt{x}\right]_0^4$$

$$= \left(\frac{2}{3}(4)\sqrt{4} + 2\sqrt{4}\right) - 0$$

$$= \frac{8}{3}(2) + 2(2)$$

$$= \frac{16}{3} + 4 = \frac{16+12}{3} = \frac{28}{3}$$

34. $\int_0^{\frac{\pi}{3}} \tan x \, dx$ is

(a) log 2

(b) 0

(c) log √2

(d) 2 log 2

Hint: $I = \int_0^{\frac{\pi}{3}} \tan x\,dx = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x}\,dx$

Put $\cos x = t \quad \Rightarrow -\sin x\,dx = dt$

$\Rightarrow \quad \sin x\,dx = -dt$

$\therefore \quad I = -\int_1^{\frac{1}{2}} \frac{dt}{t}$ [$\because$ when $x = 0$, $t = \cos$

0 $= 1$ when $x = \frac{\pi}{3}$, $t = \cos\frac{\pi}{3} = \frac{1}{2}$]

$= \int_{\frac{1}{2}}^{1} \frac{dt}{t} = [\log(t)]_{\frac{1}{2}}^{1}$

$= \log 1 - \log\frac{1}{2}$

$= \log 1 - (\log 1 - \log 2)$

$= 0 \quad 0 + \log 2 = \log 2$

35. Using the factorial representation of the gamma function, which of the following is the solution for the gamma function $\Gamma(n)$ when $n = 8$

(a) 5040

(b) 5400

(c) 4500

(d) 5540

Hint: $\Gamma(n)$ when $n = 8$ is 7!

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

36. Γ(n) is

(a) (n – 1)!

(b) n!

(c) n Γ(n)

(d) (n – 1) Γ(n)

Hint: $\Gamma(n) = (n-1)!$

$\Rightarrow \Gamma(1) = (1-1)! = 0! = 1$

37. Γ(1) is

(a) 0

(b) 1

(c) n

(d) n!

Hint: $\Gamma(n) = (n-1)!$

$\Rightarrow \Gamma(1) = (1-1)! = 0! = 1$

38. If n > 0, then Γ(n) is

(a) $\int_0^1 e^{-x}x^{n-1}dx$ (b) $\int_0^1 e^{-x}x^{n}dx$

(c) $\int_0^\infty e^{x}x^{-n}dx$ (d) $\int_0^\infty e^{-x}x^{n-1}dx$

Ans: (d)

39. Γ(3/2)

(a) $\sqrt{\pi}$

(b) $\sqrt{\pi}$

(c) $2\sqrt{\pi}$

(d) 3/2

Hint: $= \frac{1}{2} \times \sqrt{\pi} = \frac{\sqrt{\pi}}{2}$

40. $\int_0^\infty x^4 e^{-x} dx$ is

(a) 12

(b) 4

(c) 4!

(d) 64

Hint: $\int_0^\infty x^4 e^{-x}\, dx$

Here $a = 1, n = 4$

$[\because \int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}]$

$= \frac{4!}{(1)^5} = 4!$

UNIT: 5.

1. A partial differential equation has
   (A) one independent variable
   (B) two or more independent variables
   (C) more than one dependent variable
   (D) equal number of dependent and independent variables

Solution
The correct answer is (B).

If a differential equation has only one independent variable then it is called ordinary differential equation. A partial differential equation has two or more independent variables.

2. A solution to the partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

is
   (A) $\cos(3x - y)$
   (B) $x^2 + y^2$
   (C) $\sin(3x - y)$
   (D) $e^{-3\pi x}\sin(\pi y)$

Solution

The correct answer is (D).

We will solve this by substituting the given choices. The choice which satisfies the partial differential equation is the correct answer.

Let's start with option (A)

$$u = \cos(3x - y)$$

$$\frac{\partial u}{\partial x} = -3\sin(3x - y);\ \frac{\partial u}{\partial y} = \sin(3x - y)$$

$$\frac{\partial^2 u}{\partial x^2} = -9\cos(3x - y);\ \frac{\partial^2 u}{\partial y^2} = -\cos(3x - y)$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$-9\cos(3x - y) = -9\cos(3x - y)$$

Let's start with option (B)

$$u = x^2 + y^2$$

$$\frac{\partial u}{\partial x} = 2x;\ \frac{\partial u}{\partial y} = 2y$$

$$\frac{\partial^2 u}{\partial x^2} = 2;\ \frac{\partial^2 u}{\partial y^2} = 2$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$2 \neq 9 \times 2$$

Let's start with option (C)

$$u = \sin(3x - y)$$

$$\frac{\partial u}{\partial x} = 3\cos(3x - y); \frac{\partial u}{\partial y} = -\cos(3x - y)$$

$$\frac{\partial^2 u}{\partial x^2} = -9\sin(3x - y); \frac{\partial^2 u}{\partial y^2} = -\sin(3x - y)$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$-9\sin(3x - y) = -9\sin(3x - y)$$

Let's start with option (D)

$$u = e^{-3\pi x}\sin(\pi y)$$

$$\frac{\partial u}{\partial x} = -3\pi e^{-3\pi x}\sin(\pi y); \frac{\partial u}{\partial y} = \pi e^{-3\pi x}\cos(\pi y)$$

$$\frac{\partial^2 u}{\partial x^2} = 9\pi^2 e^{-3\pi x}\sin(\pi y); \frac{\partial^2 u}{\partial y^2} = -\pi^2 e^{-3\pi x}\sin(\pi y)$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$9\pi^2 e^{-3\pi x}\sin(\pi y) \neq -9\pi^2 e^{-3\pi x}\sin(\pi y)$$

3. The partial differential equation

$$5\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial y^2} = xy$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

Solution

The correct answer is (A).

A general second order partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, and C are functions of x and y and D is a function of x, y, u and $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

The above PDE can be rewritten as

$$5\frac{\partial^2 z}{\partial x^2}+0\frac{\partial^2 z}{\partial x\partial y}+6\frac{\partial^2 z}{\partial y^2}-xy=0$$

Depending on the value of B^2-4AC, the 2[nd] order linear PDE can be classified into three categories.

1. if $B^2-4AC<0$, it is called elliptic
2. if $B^2-4AC=0$, it is called parabolic
3. if $B^2-4AC>0$, it is called hyperbolic

In the above question,

$$A=5, B=0, C=6,$$

giving

$$B^2-4AC=0-4(5)(6)$$
$$=-120<0$$

This classifies the differential equation as elliptic.

4. The partial differential equation

$$xy\frac{\partial z}{\partial x}=5\frac{\partial^2 z}{\partial y^2}$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

Solution
The correct answer is (B).

A general second order partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, and C are functions of x and y and D is a function of x, y, u and $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

The above PDE can be rewritten as

$$0\frac{\partial^2 z}{\partial x^2}+0\frac{\partial^2 z}{\partial x\partial y}+5\frac{\partial^2 z}{\partial y^2}-xy\frac{\partial z}{\partial x}=0$$

Depending on the value of B^2-4AC, the 2nd order linear PDE can be classified into three categories.

1. if $B^2-4AC<0$, it is called elliptic
2. if $B^2-4AC=0$, it is called parabolic
3. if $B^2-4AC>0$, it is called hyperbolic

In the above question,

$A=0, B=0, C=5$.

giving

$$B^2-4AC=0-4(0)(5)$$
$$=0$$

This classifies the differential equation as parabolic.

5. The partial differential equation

$$\frac{\partial^2 z}{\partial x^2}-5\frac{\partial^2 z}{\partial y^2}=0$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

Solution
The correct answer is (C).

A general second order partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x \partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, and C are functions of x and y and D is a function of x, y, u and $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

The above PDE can be rewritten as

$$1\frac{\partial^2 z}{\partial x^2}+0\frac{\partial^2 z}{\partial x \partial y}-5\frac{\partial^2 z}{\partial y^2}=0$$

Depending on the value of B^2-4AC, the 2nd order linear PDE can be classified into three categories.

1. if $B^2-4AC<0$, it is called elliptic
2. if $B^2-4AC=0$, it is called parabolic
3. if $B^2-4AC>0$, it is called hyperbolic

In the above question,

$$A=1, B=0, C=-5,$$

giving

$$B^2-4AC=0-4(1)(-5)$$
$$=20>0$$

This classifies the differential equation as hyperbolic.

6. The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation.

$$\frac{\partial w}{\partial t}+\frac{\partial^3 w}{\partial x^3}-6w\frac{\partial w}{\partial x}=0$$

(A) linear; 3rd order
(A) nonlinear; 3rd order
(B) linear; 1st order
(C) nonlinear; 1st order

Solution
The correct answer is (B).

The partial differential equation is nonlinear because the coefficient of the derivative term $\frac{\partial w}{\partial x}$ is a function of the dependent variable, w. The equation is a 3rd order as that is the highest derivative in the partial differential equation.

.

7.. The general solution of a partial differential equation contains:

a. One arbitrary constant

b. Two arbitrary constants

c. Three arbitrary constants

d. No arbitrary constant

Hint: Consider the number of arbitrary constants in a general solution.

8. A quasilinear partial differential equation is characterized by:

a. Linear coefficients

b. Nonlinear coefficients

c. No coefficients

d. Constant coefficients

Hint: Consider the linearity of the coefficients.

9. The method of separation of variables is often used to solve:

a. Linear partial differential equations

b. Nonlinear partial differential equations

c. Ordinary differential equations

d. Exact differential equations

Hint: Recall the method used for solving certain types of PDEs.

9. The Cauchy problem for a partial differential equation typically involves specifying:

a. Initial values

b. Boundary values

c. Both initial and boundary values

d. No values

Hint: Think about what values need to be specified for a well-posed problem.

10. The characteristic curves associated with a hyperbolic partial differential equation:

a. Are straight lines

b. Are circles

c. Depend on the specific equation

d. Are parabolas

Hint: Recall the behavior of characteristic curves for hyperbolic PDEs.

11. The order of a partial differential equation is determined by the highest power of:

a. The dependent variable

b. The independent variable

c. The coefficients

d. The partial derivatives

Hint: Identify the key factor in determining the order of a PDE.

12. The method of characteristics is commonly used to solve which type of partial differential equations?

a. Elliptic

b. Parabolic

c. Hyperbolic

d. Quasilinear

Hint: Consider the behavior of characteristic curves.

13. The boundary conditions for a partial differential equation are usually specified along:

a. Lines

b. Surfaces

c. Volumes

d. Points

Hint: Consider where boundary conditions are applied.

14. The initial-boundary value problem involves specifying conditions:

a. Only at the initial time

b. Only at the boundaries

c. Both at the initial time and boundaries

d. Neither at the initial time nor boundaries

Hint: Think about the combination of initial and boundary conditions.

15. In the context of partial differential equations, the term "well-posedness" refers to:

a. The existence of a solution

b. The uniqueness of a solution

c. The stability of a solution

d. All of the above

Hint: Consider the criteria for a well-posed problem.

16. The method of finite differences is often used for:

a. Solving PDEs numerically

b. Solving PDEs analytically

c. Classifying PDEs

d. Transforming PDEs into ODEs

Hint: Think about numerical methods for solving PDEs.

17. The D'Alembert solution is associated with solving:

a. Elliptic equations

b. Parabolic equations

c. Hyperbolic equations

d. Quasilinear equations

Hint: Consider the specific type of PDE associated with the D'Alembert solution.

18. The order of a partial differential equation is determined by:

a. The highest power of the dependent variable

b. The highest power of the independent variable

c. The sum of powers of the partial derivatives

d. The coefficients of the equation

Hint: Focus on the variables and their derivatives.

19. The characteristic equation associated with hyperbolic PDEs typically involves:

a. Real and distinct roots

b. Real and repeated roots

c. Complex conjugate roots

d. Imaginary roots

Hint: Consider the nature of the characteristic roots for hyperbolic PDEs.

20. The method of separation of variables is most effective for solving PDEs with:

a. Constant coefficients

b. Linear coefficients

c. Nonlinear coefficients

d. Quasilinear coefficients

Hint: Recall the conditions under which separation of variables is often applied.

21. The Laplace transform is a technique commonly used for solving:

a. Ordinary differential equations

b. Partial differential equations

c. Both ordinary and partial differential equations

d. Integral equations

Hint: Think about the domain of application for Laplace transform.

22. The Cauchy-Kovalevskaya theorem is related to the existence and uniqueness of solutions for:

a. Elliptic equations

b. Parabolic equations

c. Hyperbolic equations

d. Quasilinear equations

Hint: Consider the specific type of PDE associated with the theorem.

23. In the context of PDEs, a singular solution is one that:

a. Does not satisfy the given boundary conditions

b. Has singularities in its domain

c. Is not differentiable

d. Violates the uniqueness condition

Hint: Think about the characteristics of singular solutions.

24. The method of characteristics is well-suited for solving:

a. Elliptic equations

b. Parabolic equations

c. Hyperbolic equations

d. Quasilinear equations

Hint: Recall the type of PDEs for which the method of characteristics is applicable.

25. The Green's function is often used for solving:

a. Elliptic equations

b. Parabolic equations

c. Hyperbolic equations

d. Quasilinear equations

Hint: Identify the type of PDE for which Green's function is a common tool.

26. The compatibility conditions for a system of linear partial differential equations ensure:

a. Existence of solutions

b. Uniqueness of solutions

c. Stability of solutions

d. Convergence of solutions

Hint: Consider the conditions that guarantee well-posedness.

27. The method of characteristics is particularly useful for solving PDEs in:

a. Cartesian coordinates

b. Polar coordinates

c. Cylindrical coordinates

d. Spherical coordinates

Hint: Consider the coordinate systems where the method of characteristics is applicable.

28. The general solution of a homogeneous linear second-order PDE is a linear combination of:

a. Exponential functions

b. Trigonometric functions

c. Polynomial functions

d. Logarithmic functions

Hint: Consider the types of functions commonly present in the general solution.

29. The Cauchy-Riemann equations are associated with:

a. Parabolic PDEs

b. Hyperbolic PDEs

c. Elliptic PDEs

d. Quasilinear PDEs

Hint: Think about the type of PDEs related to complex analysis.

30. The general solution of a second-order homogeneous linear PDE with constant coefficients can be expressed using:

a. Exponential functions

b. Sine and cosine functions

c. Polynomial functions

d. Logarithmic functions

Hint: Consider the characteristic equation and its solutions.

31. The Green's function for a PDE is often used to solve:

a. Boundary value problems

b. Initial value problems

c. Eigenvalue problems

d. Singular value problems

Hint: Consider the type of problems for which Green's functions are applied.

32. The solution to the Laplace equation in polar coordinates is often expressed using:

a. Sine and cosine functions

b. Bessel functions

c. Exponential functions

d. Legendre polynomials

Hint: Consider the special functions commonly used in polar coordinates.

33. The general solution of a second-order linear homogeneous PDE with constant coefficients is often a combination of:

a. Polynomials

b. Exponentials

c. Trigonometric functions

d. All of the above

Hint: Think about the types of functions that appear in the general solution.

34. The general solution of a linear first-order PDE can be expressed using the method of characteristics as:

a. $u(x, y) = f(x - y)$

b. $u(x, y) = f(x + y)$

c. $u(x, y) = f(xy)$

d. $u(x, y) = f(x) + g(y)$

Hint: Consider the characteristic curves and their role in the method of characteristics.

33. The formula for the solution of the inhomogeneous wave equation $u_{tt} = c^2u_{xx} + f(x, t)$ with initial conditions $u(x, 0) = g(x)$ and $u_t(x, 0) = h(x)$ is given by:

a. $u(x, t) = f(x + ct) + g(x - ct)$

b. $u(x, t) = f(x - ct) + g(x + ct)$

c. $u(x, t) = f(x - ct)g(x + ct)$

d. $u(x, t) = f(x + ct)g(x - ct) + g(x) + ct$

Hint: Consider the superposition principle and the form of the solution.

34. The general solution of the Helmholtz equation $u_{xx} + u_{yy} + k^2u = 0$ in Cartesian coordinates can be expressed as a combination of:

a. Sine and cosine functions

b. Bessel functions

c. Exponential functions

d. Legendre polynomials

Hint: Consider the form of solutions for the Helmholtz equation.

35. The general solution to a quasilinear PDE can often be found using the method of:

a. Separation of variables

b. Characteristics

c. Laplace transform

d. Fourier series

Hint: Consider the method commonly used for solving quasilinear PDEs.

36. The general solution to a homogeneous linear PDE can be expressed as a:

a. Particular solution

b. Sum of solutions

c. Product of solutions

d. Linear combination of solutions

Hint: Think about the structure of the general solution for homogeneous linear PDEs.

37. The partial differential equation of $z = (x - a)^2 + (y - b)^2$

(a) 4z=p2+q2
(b) z=p2+q2
(c) 4z=p2-q2
(d) none of these

Answer:
Option **(a)**

38. Eliminating the arbitrary function from z=fx2+y2 , then the partial differential equation formed is _____ .

(a)py=qx
(b)px=qy
(c)pq=xy
(d)none of these

Answer:
Option **(a)**

39.Eliminating arbitrary constants a and b from z=x+a y+b , the partial differential equation formed is ___

(a) z=p+q
(b) z=pq2
(c) z=pq
(d) none of these

Answer:
Option **(c)**

40. The partial differential equation pq=4z is _____ .

(a) linear
(b) non-linear
(c) higher order
(d) non of these

Answer:
Option **(b)**

41. The partial differential equation z=px+qy+fp, q is known as _____ .

(a) Lagrange's partial differential equation
(b) Clairaut's partialdifferential equation
(c) higher order partial differential equation
(d) none of these

Answer: Option **(b)**

MS.S.REZVANA PARVEEN WAS BORN IN 2000 IN MADURAI, TAMIL NADU AND SHE IS CURRENTLY WORKING AS AN ASSITANT PROFESSOR IN THE DEPARTMENT OF MATHEMATICS . ST.JOSEPH'S COLLEGE OF ARTS AND SCIENCE FOR WOMEN,HOSUR. SHE HAS COMPLETED HER M.SC DEGREE IN ST.JOSEPH'S COLLEGE OF ARTS AND SCIENCE FOR WOMEN ,HOSUR.SHE HAS PUBLISHED 5 INTERNATIONAL CONFERENCE PROCEEDINGS. HER AREA OF INTEREST INCLUDING NUMERICAL METHODS

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