# COMPLEX ANALYSIS

Mrs. M. MEENAKSHI., M.Sc., MPhil., (Ph.d)

## UNIT-I

1. Which of the following regions in the complex plane is simply connected?

a. Annulus b. Disk

c. Union of two disks d. Half-plane

2. What is the image of the unit circle under the mapping f(z) = z 2 ?

a. Circle centered at the origin b. Parabola

c. Ellipse d. Cardioid

3. In the complex plane, what does the set of points {z ∈ C : |z| < 1} represent?

a. Circle of radius 1 b. Circle of radius 2

c. Disk of radius 1 d. Annulus of radius 1

Hint: Focus on the inequality |z| < 1 and its geometric interpretation.

4. Which of the following functions is not holomorphic (analytic) in any region of the complex plane

a. f(z) = e z b. f(z) = z¯

c. f(z) = sin(z) d. f(z) = 1/z

Hint: Consider the Cauchy-Riemann equations for holomorphic functions.

5. What is the residue of 1/(z^2+1) at z = i?

a. 0 b. (1 )/2i

c.- (1 )/2i d. 1

Hint: Use the formula for the residue at a simple pole.

6. The function $f(z) = 1/(z-2)+1/(z+2)$ has poles at which points in the complex plane?

a. $z = 2$ b. $z = -2$

c. $z = \pm 2$ d. $z = 0$

Hint: Identify the points where the denominator becomes zero.

7. Which of the following statements is true regarding a branch cut in the complex plane?

a. It is a path connecting two branch points.

b. It is a continuous curve connecting two singularities.

c. It is a line connecting two critical points.

d. It is a curve connecting two poles.

Hint: Consider the purpose and location of a branch cut in complex analysis

8. What is the image of the upper half-plane under the mapping $f(z) = e^z$?

a. Right half-plane b. Upper half-plane

c. Unit disk d. Left half-plane

Hint: Examine the behavior of the exponential function on the complex plane.

9. If $f(z) = z^3+2z^2+1$ is a polynomial, how many zeros does it have in the complex plane?

a. 0 b. 1

c. 2 d. 3

Hint: Recall the Fundamental Theorem of Algebra.

10. The function $f(z) = \sinh(z)$ maps the imaginary axis to what curve in the complex plane?

a. Parabola b. Ellipse
c. Hyperbola d. Circle
Hint: Understand the behavior of hyperbolic functions.

11. Evaluate limz→1(z^2-1)/(z-1).
a. 0 b. 1
c. 2 d. Does not exist
Hint: Factorize the numerator and simplify.

12. What is limz→0sinz/z?
a. 0 b. 1
c. π 2 d. Does not exist
Hint: Apply L'Hopital's Rule or use the standard limit lim z→0 sinz/z?= 1.

1 3. For f(z) =(z^2-4)/(z-2), what is limz→2 f(z)?
a. 0 b. 2
c. 4 d. Does not exist
Hint: Factorize the numerator and simplify.

14. Determine lim z→i (z 2 + 1).
a. 0 b. -1
c. 2i d. Does not exist
Hint: Substitute z = i into the expression.

15. Question: What is limz→∞(3z^2-2z+1)/(z^2+1)?
a. 0 b. 3
c. 2 d. Does not exist
Hint: Divide both the numerator and denominator by the highest power of z in the expression.

16. Evaluate $\lim_{z\to 1} (z^3-1)/(z-1)$

a. 0 b. 1

c. 2 d. Does not exist

Hint: Factorize the numerator and simplify.

17. For $f(z) = 1/z$, find $\lim_{z\to\infty} f(z)$.

a. 0 b. 1

c. $\infty$ d. Does not exist

Hint: Examine the behavior of f(z) as z approaches infinity.

18. Question: Determine $\lim_{z\to 2i} (z^2-4i)$

a. -4 b. -2

c. 0 d. Does not exist

Hint: Substitute z = 2i into the expression.

19. What is $\lim_{z\to 0} e^{1/z}$?

a. 0 b. 1

c. $\infty$ d. Does not exist

Hint: Analyze the exponential behavior as z approaches 0.

20. Question: For $f(z) = \sin z/z$, find $\lim_{z\to\infty} f(z)$.

a. 0 b. 1

c. $\pi$ 2 d. Does not exist

Hint: Apply L'Hospital's Rule or use the standard limit $\lim_{x\to\infty} \sin x/x = 0$.

21. Find $\lim_{z\to\infty}(2z^2-z+1)/(z^2+3)$.

a. 0 b. 2

c. $\infty$ d. Does not exist

Hint: Divide both numerator and denominator by the highest power of z.

22. Determine limz→∞ 〚((z^2-1)/(z^2+1))〛 ^3.

a. 0 b. 1

c. $\infty$ d. Does not exist

Hint: Simplify the expression inside the parentheses.

23. Evaluate limz→∞ sin z/ z

a. 0 b. 1

c. π 2 d. Does not exist

Hint: Use the standard limit limx→0 sin x/ x = 1.

24. For f(z) =z/(z+1), find limz→∞ f(z).

a. 0 b. 1

c. $\infty$ d. Does not exist

Hint: Divide both numerator and denominator by the highest power of z.

25. Question: What is limz→01/z?

a. 0 b. 11

c. $\infty$ d. Does not exist

Hint: Analyze the behavior of 1 z as z approaches 0.

26. Determine limz→∞e^2z/(e^z+1).

a. 0 b. 1

c. $\infty$ d. Does not exist

Hint: Divide both numerator and denominator by the highest power of e z .

27. Find limz→i(z^2+1)/(z-i).

a. 0 b. 1

c. i d. Does not exist

Hint: Rationalize the denominator.

28. Evaluate limz→∞ 〖(1+2/z)〗^z

a. 0 b. 1

c. e 2 d. Does not exist

Hint: Recognize the form as the definition of e.

29. Question: For f(z) = (z^2+2z+1)/(z^2-1), find limz→−1 f(z).

a.0 b. 1

c. ∞ d. Does not exist

Hint: Factorize the numerator and simplify

30. Determine limz→2i(z^2+4i)/(z-2i).

a. 0 b. −2i

c. ∞ d. Does not exist

Hint: Rationalize the denominator.

31. If f(z) = u(x, y)+iv(x, y) is differentiable at a point z = x+iy, what condition does f(z) satisfy?

a. Cauchy-Riemann equations

b. Laplace's equation

c. Fermat's theorem
d. Mean Value Theorem for Integrals
Hint: The differentiability of f(z) implies a certain relationship between its partial derivatives

32. What are the Cauchy-Riemann equations for a complex function $f(z) = u(x, y) + iv(x, y)$?
a. $\partial u/(\partial x) = \partial v/(\partial y)$ and $\partial u/(\partial y) = -\partial v/(\partial x)$
b. $\partial u/(\partial x) = \partial v/(\partial x)$ and $\partial u/(\partial y) = \partial v/(\partial y)$
c. $\partial u/(\partial x) = \partial u/\partial y$ and $\partial v/(\partial x) = \partial v/(\partial y)$
d. $\partial u/(\partial x) = \partial v/(\partial y)$ and $\partial u/(\partial y) = \partial v/(\partial x)$
Hint: Recall the specific form of the Cauchy-Riemann equations.

33. If $f(z) = z 2$ , where $z = x+iy$, what are the partial derivatives ux and vy for f?
a. 2x and −2y        b. 2x and 2y
c. x and −y        d. x and y

Hint: Compute the partial derivatives of u and v for $f(z) = z 2$ .

3 4. For the function $f(z) = e ix$, where $z = x + iy$, what are the Cauchy-Riemann equations?
a. $\partial u/(\partial x) = \partial v/(\partial y)$ and $\partial u/(\partial y) = -\partial v/(\partial x)$
d. $\partial u/(\partial x) = \partial v/(\partial x)$ and $\partial u/(\partial y) = \partial v/(\partial y)$
c. $\partial u/(\partial x) = \partial u/\partial y$ and $\partial v/(\partial x) = \partial v/(\partial y)$
d. $\partial u/(\partial x) = \partial v/(\partial y)$ and $\partial u/(\partial y) = \partial v/(\partial x)$
Hint: Express f(z) in terms of u and v and apply the Cauchy-Riemann equations.

35. If f(z) = 1/ z , where z = x+iy, what are the partial derivatives ux and vy for f?

a. y/(x^2+y^2 ) and x/(x^2+y^2 )

b. x/(x^2+y^2 ) and y/(x^2+y^2 )

c. y/(x^2-y^2 ) and x/(x^2-y^2 )

d. x/(x^2-y^2 ) and y/(x^2-y^2 )

Hint: Express f(z) in terms of u and v and compute the partial derivatives.

36. If f(z) = e xy cos(y) + iexy sin(y), where z = x + iy, is f differentiable?

a. Yes b. No

c. Depends on x and y d. Cannot be determined

Hint: Check if the Cauchy-Riemann equations are satisfied.

37. Question: If f(z) = x 3 − 3xy2 + i(3x2y − y3 ), where z = x + iy, is f differentiable?

a. Yes b. No

c. Depends on x and y d. Cannot be determined

Hint: Check if the Cauchy-Riemann equations are satisfied.

38. Consider the function f(z) = (z^2-1)/(z-1). Where is f(z) differentiable?

a. z = 1 b. z = −1

c. z = 0 d. z is differentiable everywhere

Hint: Examine the points where f(z) might not be differentiable.

39. For f(z) = sin(x) cosh(y) + i cos(x) sinh(y), where z = x + iy, is f differentiable?

a. Yes b. No

c. Depends on x and y d. Cannot be determined

Hint: Check if the Cauchy-Riemann equations are satisfied.

40. Question: If f(z) =z¯/z, where z = x + iy, is f differentiable?

a. Yes b. No

c. Depends on x and y d. Cannot be determined

Hint: Express f(z) in terms of u and v and check the Cauchy-Riemann equations.

41. If a complex function f(z) = u(x, y) +iv(x, y) is differentiable at a point z = x + iy, what is a sufficient condition?

a. ux = vy and uy = −vx b. ux = vx and uy = vy

c. ux = uy and vx = vy d. ux = vy and uy = vx

Hint: Recall the Cauchy-Riemann equations for differentiability.

42. If f(z) = reiθ in polar coordinates, what is the expression for f ′ (z)?

a. e iθ b. r · e iθ

c. ir · e iθ d. i · e iθ

Hint: Differentiate f(z) with respect to z in polar coordinates.

43. For f(z) = e ix, where z = reiθ in polar coordinates, what are the polar form derivatives f ′ (z)?

a. e iθ b. ireiθ

c. reiθ d. i · e iθ

Hint: Express f(z) in terms of r and θ and differentiate.

44. If a complex function f(z) is analytic in a region D, what is a sufficient condition for f(z) to be differentiable at every point in D?

a. The Cauchy-Riemann equations are satisfied.

b. f(z) is continuous in D.

c. f(z) has a continuous derivative in D.

d. f(z) is holomorphic in D.

Hint: Consider the definition of an analytic function.

45. If f(z) = 1/ z , where z = reiθ in polar coordinates, what is the expression for f ′ (z)?

a. – 1/(r^2 e^iθ ) b. 1/(re^iθ )

c. –i/(re^iθ ) d. 1/(r^2 e^iθ )

Hint: Express f(z) in terms of r and θ and differentiate.

Answer: C) –i/(re^iθ )

46. For f(z) = z 3 , where z = reiθ in polar coordinates, what is the expression for f ′ (z)?

a. 3r 2 e 3iθ b. 3reiθ

c. 3r 2 e iθ d. 3re3iθ

Hint: Express f(z) in terms of r and θ and differentiate.

47. If f(z) =(z^2-1)/(z-1), what is the sufficient condition for f(z) to be differentiable at every point in its domain?

a. z = 1 is not in the domain of f(z).

b. f(z) is continuous at z = 1.

c. f(z) is differentiable at z = 1

d. f(z) is holomorphic everywhere.

Hint: Check if z = 1 is a removable singularity.

48. If f(z) = ln(r) +iθ, where z = reiθ in polar coordinates, what is the expression for f ′ (z)?

a. 1/r-I b. 1/r+i

c. -1/r-i c. -1/r+i

Hint: Express f(z) in terms of r and θ and differentiate.

49. If f(z) = e z , where z = reiθ in polar coordinates, what is the expression for f ′ (z)?

a. e r e iθ b. irer e iθ

c. e reiθ d. e r

Hint: Express f(z) in terms of r and θ and differentiate.

50. If f(z) = (sin(r))/r+iθ, where z = reiθ in polar coordinates, what is the expression for f ′ (z)?

a. (cos(r))/r+ i b. (cos(r))/r− i

c. (sin(r))/r− i d. (sin(r))/r+ i

Hint: Express f(z) in terms of r and θ and differentiate.

51. What is a necessary condition for a complex function f(z) to be analytic in a region?

a. Continuous partial derivatives

b. Existence of a limit

c. Satisfying the Cauchy-Riemann equations

d. Being holomorphic

Hint: Think about the fundamental requirement for a function to be analytic.

52. Which of the following functions is analytic in the entire complex plane?

a. f(z) = sin(z) b. f(z) = e z + ¯z

c. f(z) = 1 z−1 d. f(z) = ln(z)

Hint: Consider the conditions for a function to be analytic everywhere.

53. If f(z) = z 2 + 3z + 2, is f(z) analytic in the entire complex plane?

a. Yes b. No

c. Depends on the region d. Cannot be determined

Hint: Check if f(z) satisfies the conditions for analyticity.

54.: Which of the following functions is harmonic in the entire complex plane?

a. u(x, y) = e x cos(y) b. u(x, y) = x 2 − y 2

c. u(x, y) = sin(x) cosh(y) d. u(x, y) = ln(x 2 + y 2 )

Hint: Recall the definition of harmonic functions.

55. If f(z) =1/(z^2+1), is f(z) analytic in the entire complex plane? 1

a. Yes b. No

c. Depends on the region c. Cannot be determined

Hint: Investigate the poles and singularities of f(z).

56. Which of the following functions is not analytic anywhere in the complex plane?

a. $f(z) = e^{-z}$ b. $f(z) = \sqrt{z}$
c. $f(z) = \cos(\bar{z})$ d. $f(z) = \ln(|z|)$
Hint: Consider the conditions for a function to be analytic.

57. If $f(z) = e^{z} + \sin(z)$, is f(z) analytic in the entire complex plane?
a. Yes b. No
c. Depends on the region d. Cannot be determined
Hint: Check if f(z) satisfies the conditions for analyticity.

58. Which of the following functions is analytic on the unit disk $|z| < 1$?
a. $f(z) = 1/(z+1)$ b. $f(z) = \sqrt{z}$
c. $f(z) = e^{iz}$ d. $f(z) = \ln(|z|)$
Hint: Consider the radius of convergence for each function.

59. If $f(z) = z/(z-1)$, is f(z) analytic in the entire complex plane?
a. Yes b. No 2
c. Depends on the region c. Cannot be determined
Hint: Investigate the poles and singularities of f(z).

60. Which of the following functions is not harmonic in any region of the complex plane?
a. $u(x, y) = x^2 - y^2$ b. $u(x, y) = \sin(x)$
c. $u(x, y) = e^{x} \cos(y)$ d. $u(x, y) = \ln(x^2 + y^2)$
Hint: Recall the properties of harmonic functions.

**ANSWERS**

1.b, 2.d, 3.c, 4.b, 5.c, 6.c, 7.a, 8.a, 9.d, 10.c, 11.c, 12.b, 13.b, 14.b, 15.b, 16.c, 17.a,18.a, 19.d, 20.a, 21.b, 22.b, 23.b, 24.a, 25.c, 26.a, 27.c, 28.c, 29.c, 30.b, 31.a, 32.d, 33.b, 34.b, 35.a, 36.a, 37.a, 38.b, 39.b, 40.b, 41.a, 42.c, 43.b, 44.d, 45.c, 46.d, 47.a, 48.a, 49.a, 50.a, 51.c, 52.a, 53.a, 54.b, 55.a, 56.d, 57.a, 58.c, 59.b, 60.b.

## UNIT-II

1.What is the derivative of the complex function \(f(z) = e^{iz}\)?

a. i cos (z) b. –i sin(z)

c. - i cos (z) d. i sin(z)

Hint: Use the chain rule and the derivative of $e^u$

2. Consider the function f(z)= $\frac{1}{z}$ $for\ z \neq 0$ What is the residue of f(z) at z=0?

a. 0 b. 1

c. -1 d. $\frac{1}{z}$

Hint: Identify the pole at z=0 and recall the formula for the residue.

3. For a closed contour C, if the functionf(z) is analytic inside C and on C what can be said about the contour integral $\oint_C f(z)dz$?

a. It is always zero

b. It depends on the shape of C

c. It is non-zero if f(z) has a pole inside C

d. It is non-zero if f(z)has a singularity on C

Hint: Recall a fundamental property of analytic functions and contour integrals.

4. What is the derivative of the complex conjugate g(z)= $\overline{z}$

a. 1 b. $\bar{z}$

c. -1 d. - $\bar{z}$

5. If f(z) is analytic in a domain D, what is the relationship between the contour integral $\int_C f(z)$ and the antiderivative of f(z) in D?
a. They are always equal
b. They are equal only if \(C\) is a closed curve
c. They are equal only if \(f(z)\) is real-valued
d. They are never equal
Hint: Recall a property related to the antiderivative of analytic functions.

6. What is the residue of the function f(z)=$\frac{1}{z^2(z-1)}$ at the pole z = 1?

a. 0
b. $\frac{1}{z^2}$
c. -$\frac{1}{z^2}$
d. $\frac{1}{z}$

Hint: Determine the order of the pole at z = 1 and apply the residue formula.

7. If f(z) is entire (analytic over the entire complex plane), what can be said about its contour integrals over closed curves
a. They are always zero
b. They are always non-zer
c. They are only zero for circles
d. They are only zero for straight-line segments
Hint: Think about the properties of entire functions.

8. What is the residue of the function f(z) = $\frac{sinz}{z^2}$ at the pole z=0?

a. 0
b. 1
c. ½
d. -1/2

Hint: Consider the Taylor expansion of sin z

9. If (z) is analytic on and inside a simple closed contour C, and f(z) has a simple pole at z = a inside C, what is the value of $\oint_C (z - a)f(z)dz$?

a. 0　　b. $2\pi i$

c. -$2\pi i$　　d. $2\pi$

Hint: Use the residue theorem.

10. What is the derivative of the complex function f(z) =In $(z^2 + 1)$?

a. $\frac{2z}{z^2+1}$

b. $\frac{2z}{(z^2+1)2}$

c. $\frac{1}{z^2+1}$

d. $\frac{1}{z^2+1}$ multiplied by the derivative of ln(u).

Hint: Apply the chain rule and the derivative of ln(u).

11. Evaluate the contour integral $\oint_C (z^2 + 3z)dz$ , Where C is the circle |
z| = 2.

a. 0　　b. $12\pi i$

c. $24\pi i$　　d. $4\pi i$

Hint: Use the Cauchy Integral Formula.

12. Find the antiderivative of f(z)=$e^{2z}$+cos(3z)

a. $e^{2z} + \frac{1}{3}\sin(3z) + C$

b. $e^{2z} - \frac{1}{3}\sin(3z) + C$

c. $e^{2z} + \frac{1}{3}\cos(3z) + C$

d. $e^{2z} - \frac{1}{3}\cos(3z) + C$

Hint: Use the power rule for exponential functions and trigonometric functions.

13. Calculate $\int_C \frac{1}{z^2+1}$ where C is the semicircle in the upper half-plane centered at the origin with radius \(2\).

a. $\pi$ b. $\frac{\pi}{2}$

c. $2\,\pi$ d. $\frac{\pi}{4}$

Hint: Use the residue theorem.

14. Determine the value of $\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2+1}$dx

a. $\pi$ b. $\frac{\pi}{2}$

c. $2\,\pi$ d. 0

Hint: Consider the contour integral on a semicircle in the upper half-plane.

15. Find the Laurent series expansion of f(z) = $\frac{1}{z^2-4}$centered at \(z = 0\).

a. $\frac{1}{4} - \frac{1}{2z} + \frac{1}{4z^2}$

b. $\frac{1}{4} + \frac{1}{2z} + \frac{1}{4z^2}$

c. $-\frac{1}{4} - \frac{1}{2z} - \frac{1}{4z^2}$

d. $-\frac{1}{4} + \frac{1}{2z} + \frac{1}{4z^2}$

Hint: Factor the denominator and use partial fraction decomposition.

16. Evaluate $\int_C \frac{sinz}{z} dz$ Where C is the unit circle centered at the origin.

a. 0 b. $\pi$

c. $\frac{\pi}{2}$ d. 2 $\pi$

Hint: Use the Cauchy's Integral Formula for derivatives.

17. Determine the poles and residues of the function f(z)=$\frac{1}{z^2+2z}$

a. Pole at z=0 , , Residue 1

b. Poles at z=0, z=-2, Residues 0 &1

c. Pole at z = -2, Residue 0

d. Poles at z = 0 and z = -2, Residues 1 and 0

Hint: Identify the singularities and calculate the residues.

18. Find the value of $\int_{-\infty}^{\infty} \frac{cos(2x)}{x^2+1}$ dx

a. π b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$ d. 0

Hint: Use a semicircular contour in the upper half-plane.

19. Compute the integral $\int_{-\infty}^{\infty} \frac{x}{x^2+4}$.

a. π b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$ d. 0

Hint:Consider the semicircular contour in the upper half-plane.

20. Find the value of $\int_0^{2\pi} e^{i\theta}\, d\theta$.

a. 0 b. $2\pi$
c. i d. -i

Hint:Parametrize the contour using z= $e^{i\theta}$

21. Which of the following statements is a consequence of the Cauchy-Goursat Theorem?

a. If f(z) is holomorphic on a disk, then it is analytic on the disk.

b. If f(z) has a pole inside a closed contour, the integral over the contour
is zero.

c. If f(z) is analytic in a simply connected region, then its contour integral
is independent of the path.

d. If f(z) is holomorphic, then its derivative is holomorphic.

Hint:Recall the key properties of the Cauchy-Goursat Theorem.

22.What is a key requirement for a domain to be simply connected?

a. The domain has no singularities.
b. The domain is connected and has no holes.
c. The domain is a closed disk.
d. The domain has a removable singularity.

Hint:Think about the connectivity of the domain.

23. In the context of complex analysis, what does the Cauchy Integral Formula express?

a. The derivative of a holomorphic function.
b. The value of a contour integral for analytic functions.
c. The behavior of a function near a singularity.
d. The sum of residues inside a closed contour.

Hint:Consider the formula and its applications.

24. What is the main idea behind the proof of the Cauchy-Goursat Theorem?

a. Residue calculus.
b. Analytic continuation.
c. Green's Theorem.
d. The Cauchy Integral Formula.
Hint:Think about integrating over a simply connected region.

25. What is the primary use of the Cauchy Residue Theorem?
a. To calculate the value of contour integrals involving singularities.
b. To find the poles of a complex function.
c. To determine if a function is holomorphic.
d. To evaluate improper integrals.
Hint:Consider where the residues are located.

26. In a multiply connected domain, how many paths are needed to evaluate a contour integral?
a. One path is always sufficient.
b. It depends on the complexity of the domain.
c. Multiple paths are required.
d. A straight-line path is the only requirement.
Hint: Think about the nature of multiply connected domains.

27. Which statement about a simply connected domain is true?
a. Every closed curve in the domain can be continuously shrunk to a point.

b. There exists a closed curve that cannot be continuously shrunk to a point.
c. Every closed curve in the domain contains a singularity.
d. The domain must be a closed disk.
Hint:Consider the topological properties of simply connected domains.

28. What is a necessary condition for a function to be holomorphic?
a. The function must be continuous.
b. The Cauchy-Riemann equations must be satisfied.
c. The function must be differentiable.
d. The domain must be a closed disk.
Hint: Recall the conditions for a function to be holomorphic.

29. What does the winding number of a closed curve in a simply connected domain indicate?
a. The number of times the curve intersects itself.
b. The number of times the curve encircles a singularity.
c. The number of holes inside the curve.
d. The length of the curve.
Hint: Consider the concept of winding number in the context of simply connected domains.

30. In a multiply connected domain, what can be said about the number of poles of a holomorphic function?
a. There are no poles in multiply connected domains.
b. The number of poles equals the number of holes in the domain.
c. The number of poles is always infinite.
d. The number of poles is unrelated to the topology of the domain.
Hint: Relate the concept of poles to the topology of the domain.

31. What does the Cauchy Integral Formula state?
a. It expresses the relationship between holomorphic and analytic functions.
b. It provides a formula for the derivative of a complex function.
c. It relates the values of a holomorphic function inside a closed contour to its values on the contour itself.
d. It is a generalization of the Fundamental Theorem of Calculus.
Hint: Consider the purpose of the Cauchy Integral Formula.

32. What is the formula for the Cauchy Integral Formula?
a. $\oint_C f(z)dz = f'(z_0)$
b. $\oint_C f(z)dz = f(z_0)$
c. $\oint_C f(z)dz = 2\pi i(z_0)$
d. $\oint_C f(z)dz = 2\pi i f'(z_0)$
Hint: Recall the structure of the Cauchy Integral Formula.

33. What is the condition for the Cauchy Integral Formula to hold for a closed contour \(C\)?
a. The function must be continuous on C.
b. The function must be holomorphic inside and on C.
c. The contour must be a simple closed curve.
d. The contour must be a circle.
Hint: Think about the requirement for the function and the contour.

34. What is the purpose of the Cauchy Residue Theorem?
a. To find the residues of a function.
b. To evaluate contour integrals without calculating residues.

c. To determine if a function is holomorphic.
d. To calculate the value of contour integrals involving singularities.
Hint: Consider the use of the Cauchy Residue Theorem.

35. How does the Cauchy Integral Formula change when applied to a contour C that encloses multiple poles?
a. The formula remains the same.
b. The formula is multiplied by the sum of residues inside C.
c. The formula is divided by the sum of residues inside C.
d. The formula becomes undefined.
Hint: Consider the effect of multiple poles on the formula.

36. What is an extension of the Cauchy Integral Formula that applies to a function on a simply connected domain?
a. Cauchy's Residue Theorem.
b. Morera's Theorem.
c. Cauchy's Integral Theorem.
d. Liouville's Theorem.
Hint: Consider theorems that involve the entire simply connected domain.

37. When does the Extended Cauchy Integral Formula (for a simply connected domain) hold?
a. When the function is holomorphic on the boundary of the domain.
b. When the function is continuous inside the domain.
c. When the function is holomorphic inside the domain.
d. When the domain is multiply connected.
Hint:Think about the requirements for the Extended Cauchy Integral Formula.

38.What is the formula for the Extended Cauchy Integral Formula?

a. $\oint_c f(z)dz = 2\pi i f'(z_0)$

b. $\oint_c f(z)dz = f(z_0)$

c. $\oint_c f(z)dz = \frac{1}{2\pi i}\oint_c \frac{f(z)}{z-z_0}$ dz

d. $\oint_c f(z)dz = \frac{1}{2\pi i}\oint_c \frac{f'(z_0)}{z-z_0}$ dz

Hint: Recall the structure of the Extended Cauchy Integral Formula.

39. What is the purpose of Morera's Theorem?

a. To determine if a function is holomorphic.

b. To find the residues of a function.

c. To evaluate improper integrals.

d. To extend the Cauchy Integral Formula to simply connected domains.

Hint: Consider the conditions under which Morera's Theorem applies.

40. In the context of the Cauchy Integral Formula, what does f'($z_0$) represent?

a. The residue of the function at $z_0$.

b. The derivative of f(z) at $z_0$

c. The value of f(z) at $z_0$.

d. The winding number of the contour around $z_0$.

Hint: Relate f'($z_0$) to the function f(z)

41. What does Liouville's Theorem state?

a. Every holomorphic function is constant.

b. Every entire function has a removable singularity.

c. Every bounded holomorphic function is constant.

d. Every meromorphic function is analytic.

Hint: Consider the behavior of holomorphic functions.

42. What is the primary implication of Liouville's Theorem?
a. Holomorphic functions can only have isolated singularities.
b. Bounded entire functions are constant.
c. Meromorphic functions have poles.
d. Analytic functions are necessarily holomorphic.
Hint: Focus on the consequence of Liouville's Theorem.

43. Which of the following functions violates Liouville's Theorem?
a. f(z)= $e^z$ b. f(z)= sin(z)
c. f(z)= $\frac{1}{z}$ d. f(z) cos(z)
Hint: Think about the conditions for Liouville's Theorem.

44. In the context of Liouville's Theorem, what is meant by a "bounded" function?

a. The function is defined over a bounded domain.
b. The values of the function are restricted to a bounded range.
c. The function has a finite number of singularities.
d. The function is holomorphic on the entire complex plane.
Hint: Consider the meaning of "bounded" in the context of functions.

45. What is the Fundamental Theorem of Algebra?
a. Every polynomial of degree n has n roots.
b. Every entire function is a polynomial.
c. Every holomorphic function has a root.
d. Every polynomial of degree n has at least one root in the complex plane.
Hint: Focus on the nature of roots in polynomials.

46. Which of the following is a consequence of the Fundamental Theorem of Algebra?
a. Every entire function is a polynomial.
b. Every polynomial can be factored into linear factors.
c. Every meromorphic function has a removable singularity.
d. Every polynomial has a finite number of roots.
Hint: Think about the factorization of polynomials.

47. How does the Fundamental Theorem of Algebra relate to the zeros of a polynomial?
a. It guarantees the existence of at least one zero in the complex plane.
b. It provides the total number of zeros of a polynomial.
c. It determines the multiplicity of each zero.
d. It ensures that every zero is real.
Hint:Consider the focus of the Fundamental Theorem of Algebra.

48. Which of the following statements is true regarding Liouville's Theorem?
a. Bounded holomorphic functions are necessarily constant.
b. Bounded holomorphic functions have at most one singularity.
c. Unbounded holomorphic functions are constant.
d. Liouville's Theorem only applies to entire functions.
Hint: Pay attention to the specific conditions of Liouville's Theorem.

49. In the Fundamental Theorem of Algebra, what is the role of the degree
of a polynomial?
a. It determines the location of the roots in the complex plane.
b. It specifies the number of terms in the polynomial.

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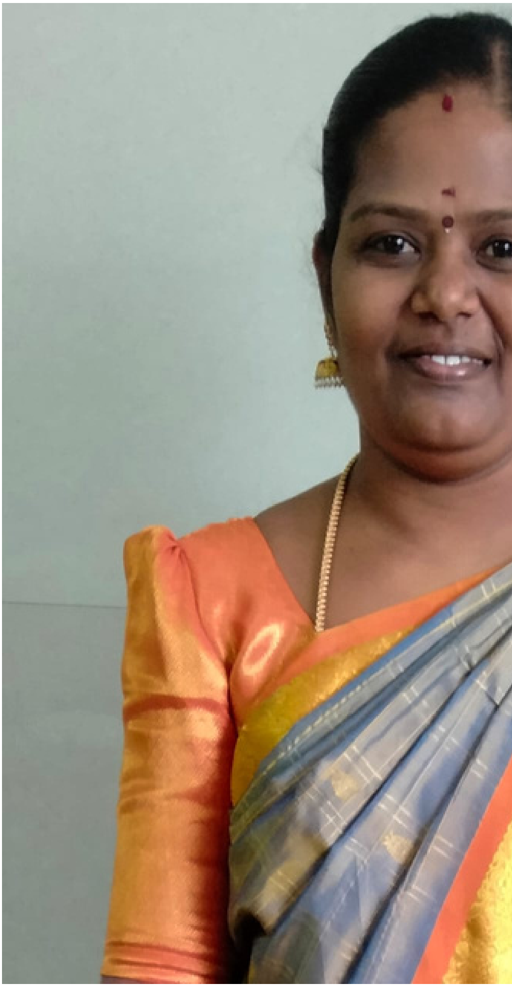

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a. The series converges absolutely.
b. The series converges conditionally.
c. The series diverges.
d. The convergence cannot be determined without additional information.

Hint: Think about the behavior of a series with positive terms.

9. What is the condition for conditional convergence of a series?

a. The series must have alternating terms.
b. The series must converge, but not absolutely.
c. The series must have complex terms.
d. The series must have a finite number of terms.

Hint: Focus on the type of convergence described.

10. If $\sum_{n=1}^{\infty} a_n$ converges absolutely and $\sum_{n=1}^{\infty} b_n$ converges conditionally, what can be said about

a. The series $\sum_{n=1}^{\infty} (a_n + b_n)$ converges absolutely.
b. The series $\sum_{n=1}^{\infty} (a_n + b_n)$ converges conditionally.
c. The series $\sum_{n=1}^{\infty} (a_n + b_n)$ may converge or diverge.
d. The series $\sum_{n=1}^{\infty} (a_n + b_n)$ diverges.

Hint: Consider the combination of absolute and conditional convergence.

11. What is the formula for the Taylor series expansion of a complex function f(z) centered at z0?

a. $\sum_{n=0}^{\infty} \frac{f^{(n)}(z_0)}{n!} (z-z_0)^n$

b. $\sum_{n=1}^{\infty} \frac{f^{(n)}(z_0)}{n!} (z_0-z)^n$

c. $\sum_{n=0}^{\infty} \frac{f^{(n)}(z)}{n!} (z-z_0)^n$

d. $\sum_{n=1}^{\infty} \frac{f^{(n)}(z)}{n!} (z-z_0)^n$

Hint: Recall the general formula for the Taylor series expansion.

12. What is the main idea behind Taylor's Theorem?

a. It provides a method to calculate the Taylor series.

b. It expresses a function as an infinite sum of its derivatives.

c. It gives an approximation of a function using its derivatives at a single point.

d. It establishes the convergence of Taylor series.

Hint: Think about the purpose and application of Taylor's Theorem.

13. What condition does a function need to satisfy for Taylor's Theorem to hold?

a. The function must be holomorphic.

b. The function must be continuous.

c. The function must be differentiable in an open interval containing the center.

d. The function must be bounded.

Hint: Consider the requirements for the application of Taylor's Theorem.

14. What is the remainder term in Taylor's Theorem called?

a. Residue
b. Error term
c. Residual
d. Lagrange term

Hint: Focus on the term used for the part not included in the Taylor series

15. In Taylor's Theorem, what is the Lagrange form of the remainder term?

a. R_n(z)=f^(n+1 (ε))/((n+1)!)(z-z0)n+1, where ε is between z & z0
b. R_n(z)=f^(n (ε))/((n+1)!)(z-z0)n+1, where ε is between z & z0
c. R_n(z)=f^(n+1 (ε))/((n+1)!)(z-z0)n+1, where ε is between z & z0
d. R_n(z)=f^(n (ε))/n!(z-z0)n+1, where ε is between z & z0

Hint: Recall the specific form of the Lagrange remainder.

16. If a function f(z) is equal to its Taylor series in some open disk, what can be concluded about f(z)?

a. f(z) is holomorphic. b. f(z) is constant.
c. f(z) is entire. d. f(z) is bounded.

Hint: Consider the relationship between the equality of a function and its Taylor series.

17. What is the Taylor series expansion of ez centered at z = 0?

a. ∑_(n=0)^∞▒z^n/n!
b. ∑_(n=1)^∞▒z^n/n!
c. ∑_(n=0)^∞▒〖〖(-1)〗^n z〗^n/n!
d. ∑_(n=1)^∞▒〖〖(-1)〗^n-1z〗^n/n!

Hint: Recall the well-known Taylor series for ez

18. What is the Taylor series expansion of sin(z) centered at z = 0?

a. ∑_(n=0)^∞▒〖(-1)〗^n/((2n+1)!) z^(2n+1)
b. ∑_(n=1)^∞▒〖(-1)〗^(n-1)/((2n-1)!) z^(2n-1)
c. ∑_(n=0)^∞▒〖(-1)〗^n/((2n)!) z^2n

d. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n)!} z^{2n}$

Hint: Recall the Taylor series expansion for sin(z).

19.What is the Taylor series expansion of 1/(1+z) centered at z = 0?

a. $\sum_{n=0}^{\infty} (-1)^n z^n$

b. $\sum_{n=1}^{\infty} (-1)^{n-1} z^n$

c. $\sum_{n=0}^{\infty} (-1)^n z^{n+1}$

d. $\sum_{n=1}^{\infty} (-1)^{n-1} z^{n+1}$

Hint: Derive the Taylor series for 1/(1+z)

20. What is the Taylor series expansion of log(1+z) centered at z = 0?

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^n}{n}$

b. $\sum_{n=0}^{\infty} \frac{(-1)^n z^{n+1}}{n+1}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^n}{n+1}$

d. $\sum_{n=0}^{\infty} \frac{(-1)^n z^{n+1}}{n}$

Hint: Consider the Taylor series expansion for log(1+z).

21.What is a Laurent series?

a. An expansion of a complex function in a power series.
b. A series of Laurent polynomials.
c. A power series with both positive and negative powers of $z$.
d. A series of Laurent integrals.
Hint: Focus on the combination of positive and negative powers.

22. What is the Laurent series expansion of a function f(z) about a point z0?

   a. $\sum_{n=-\infty}^{\infty} a_n (z-z_0)^n$

   b. $\sum_{n=1}^{\infty} a_n (z-z_0)^n$

   c. $\sum_{n=0}^{\infty} a_n (z-z_0)^n$

   d. $\sum_{n=-\infty}^{\infty} a_n (z)^n$

   Hint: Consider the inclusion of both positive and negative powers.

23. What is a laurentian singularity?

   a. A singularity that can be resolved by Laurent series expansion.
   b. A singularity with only positive powers in its expansion.
   c. A singularity that cannot be resolved by Laurent series expansion.
   d. A singularity with only negative powers in its expansion.

   Hint: Think about the nature of singularities in Laurent series.

24. What is the Laurent series representation of 1/(z(z-1)) about z = 0?

   a. $\sum_{n=-\infty}^{\infty} z^n$
   b. $\sum_{n=0}^{\infty} z^n$
   c. $\sum_{n=-\infty}^{-2} z^n$
   d. $\sum_{n=1}^{\infty} z^{-n}$

   Hint: Consider the poles and their orders.

25. What does Laurent's Theorem state?

   a. Every holomorphic function can be expressed as a Laurent series.
   b. Every meromorphic function can be expressed as a Taylor series.
   c. Every function with a singularity can be expressed as a Laurent series.

d. Every entire function can be expressed as a power series.
Hint: Focus on the relationship between functions and Laurent series.

26. What is the Laurent series expansion of 1/(z^2+z-2) about z = 1?

a. $\sum_{n=0}^{\infty} (z-1)^n$

b. $\sum_{n=-\infty}^{\infty} (z-1)^n$

c. $\sum_{n=1}^{\infty} (z-1)^{-n}$

d. $\sum_{n=-\infty}^{\infty} (z-1)^{-n-1}$

Hint: Identify the singularities and their orders.

27. In Laurent series, what is the region of convergence?
a. The entire complex plane.
b. A punctured disk centered at the singularity.
c. The annulus between the inner and outer radii of convergence.
d. The open disk centered at the singularity.
Hint: Consider the behavior around singularities.

28. What is the Laurent series expansion of 1/(z-2) about z = 1?

a. $\sum_{n=0}^{\infty} (z-1)^n$

b. $\sum_{n=-\infty}^{\infty} (z-1)^n$

c. $\sum_{n=1}^{\infty} (z-1)^{-n}$

d. $\sum_{n=-\infty}^{\infty} (z-1)^{-n-1}$

Hint: Identify the singularity and its order.

29. What is the residue of a Laurent series expansion at a singularity z=z0?

a. The coefficient of $(z-z_0)^{-1}$ in the expansion.
b. The constant term in the expansion.
c. The coefficient of $(z-z_0)^0$ in the expansion.
d. The coefficient of the highest positive power in the expansion.
Hint: Focus on the term associated with the singularity.

30. What is the Laurent series expansion of  about $z = 0$?
a. $\sum_{n=-\infty}^{\infty} z^n/n!$
b. $\sum_{n=0}^{\infty} z^n/n!$
c. $\sum_{n=-\infty}^{-2} z^n/n!$
d. $\sum_{n=1}^{\infty} z^{-n}/((n-2)!)$
Hint: Identify the singularities and their orders.

31. What is the Laurent series representation of a function $f(z)$ in an annulus $r < |z - z_0| < R$?
a. $\sum_{n=-\infty}^{\infty} a_n (z-z_0)^n$
b. $\sum_{n=0}^{\infty} a_n (z-z_0)^n$
c. $\sum_{n=-\infty}^{\infty} a_n (z-z_0)^{n+1}$
d. $\sum_{n=1}^{\infty} a_n (z-z_0)^{n-1}$
Hint: Consider the allowed range of $|z - z_0|$ in an annulus.

32. What is the Laurent series expansion of $f(z)=$  in the annulus $1 < |z| < 2$
a. $\sum_{n=-\infty}^{\infty} \frac{1}{n} z^n$
b. $\sum_{n=0}^{\infty} \frac{1}{n+1} z^n$
c. $\sum_{n=-\infty}^{-2} z^n$
d. $\sum_{n=1}^{\infty} \frac{1}{n} z^{-n}$
Hint: Identify the singularities and their corresponding powers.

33. What does Laurent's Theorem state?
   a. Every holomorphic function can be expressed as a Laurent series.
   b. Every function with a pole can be expressed as a Taylor series.
   c. Every meromorphic function can be expressed as a Laurent series.
   d. Every entire function can be expressed as a power series.
   Hint: Consider the conditions for expressing functions in Laurent series.

34. What is a removable singularity in the context of Laurent series?
   a. A singularity that cannot be resolved by a Laurent series.
   b. A singularity where the function is undefined.
   c. A singularity that can be removed by adjusting the Laurent series expansion.
   d. A singularity that is essential.
   Hint: Think about the nature of removable singularities.

35. In a Laurent series expansion, what does $a_{-1}$ represent?
   a. The constant term.
   b. The term with the highest positive power.
   c. The residue at a simple pole.
   d. The coefficient of the term with the highest negative power.
   Hint: Focus on the term associated with the principal part.

36. What is the Laurent series expansion of $e^z/(z(z-1))$ in the annulus $0 < |z| < 1$?
   a. $\sum_{n=-\infty}^{\infty} z^n/n!$
   b. $\sum_{n=0}^{\infty} z^n/n!$
   c. $\sum_{n=-\infty}^{-2} z^n/n!$
   d. $\sum_{n=1}^{\infty} z^{-n}/((n-2)!)$

Hint: Identify the singularities and their corresponding powers.

37. What is the Laurent series expansion of 1/(z^2+4) in the annulus 2 < |z| < 3?

a. ∑_(n=-∞)^∞▒〖(-1)〗^n/2 z^2n

b. ∑_(n=0)^∞▒〖(-1)〗^n/2 z^2n

c. ∑_(n=-∞)^1▒〖(-1)〗^n/2 z^2n

d.∑_(n=1)^∞▒〖(-1)〗^(n-1)/2 z^(2n-2)

Hint: Identify the singularities and their corresponding powers.

38. What is the residue at a pole of orderk in the Laurent series expansion?

a. The coefficient of (z-z0)-k

b. The constant term in the expansion.

c. The sum of the coefficients of positive powers.

d. The term with the highest positive power.

Hint: Focus on the term associated with the pole.

39.What is the Laurent series expansion of in the annulus 0 < |z| < 1?

a. ∑_(n=-∞)^∞▒〖(-1)〗^n/(n+2) z^(2+n)

b. ∑_(n=0)^∞▒〖(-1)〗^n/(n+1) z^(n+1)

c. ∑_(n=-∞)^(-3)▒〖(-1)〗^n/n z^2n

d. ∑_(n=1)^∞▒〖(-1)〗^(n-1)/n z^(n-1)

Hint: Identify the singularities and their corresponding powers.

40. What is the region of convergence for a Laurent series?

A) The entire complex plane.
B) The open disk centered at the singularity.
C) The annulus between the inner and outer radii of convergence.
D) A punctured disk centered at the singularity.
Hint: Consider the behavior around singularities.

**ANSWERS**

1.b, 2.c, 3.d, 4.b, 5.c, 6.c, 7.a, 8.a, 9.b, 10.a, 11.c, 12.c, 13.c, 14.b, 15.c, 16.c, 17.a, 18.b, 19.a, 20.a, 21.c, 22.a, 23.a, 24.c, 25.c, 26.d, 27.c, 28.c, 29.a, 30.b, 31.c, 32.c, 33.c, 34.c, 35.c, 36.c, 37.c, 38.a, 39.c, 40.c.0

## UNIT-IV

1. What is an isolated singular point of a complex function?
a. A point where the function is not defined.
b. A point where the function has an essential singularity.
c. A point surrounded by a deleted neighborhood where the function is
holomorphic.
d. A point where the function has a removable singularity.
Hint: Consider the nature of singular points.

2. At an isolated singular point, how can a function be represented as a Laurent series?
a. Only using positive powers of $(z - z_0)$.
b. Only using negative powers of $(z-z_0)$
c. Using both positive and negative powers of $(z-z_0)$

d. As a Taylor series.

Hint: Think about the structure of Laurent series at isolated singular points.

3. What is the residue of a function f(z) at an isolated singular point $(z-z_0)$ ?

a. The constant term in the Laurent series expansion.

b. The coefficient of the term with the highest positive power.

c. The coefficient of the term with the highest negative power.

d. The sum of all coefficients in the Laurent series.

Hint: Focus on the term associated with $(z-z_0)^{-1}$

4. If f(z) has a simple pole at $(z-z_0)$ , what is the residue of f(z) at that point?

a. The constant term in the Laurent series expansion.

b. The coefficient of the term with the highest positive power.

c. The coefficient of the term with $(z-z_0)^{-1}$.

d. The sum of all coefficients in the Laurent series.

Hint:For a simple pole, consider the term with $(z-z_0)^{-1}$.

5.What is the residue of $f(z)=\frac{1}{z^2+1}$ at z=i?

a. $\frac{1}{2i}$ b. $\frac{1}{2}$

c. $-\frac{1}{2i}$ d. $-\frac{1}{2}$

Hint: Identify the term with $(z - i)^{-1}$ in the Laurent series.

6. What is an essential singularity of a function?

a. A singularity that can be removed by adjusting the function's domain.

b. A singularity that cannot be removed by adjusting the function's domain.

c. A removable singularity.

d. A singularity that is not isolated.

Hint: Focus on the persistence of the singularity.

7. If a functionf(z) has a pole of order 3 at z = 2, what is the residue at that point?

a. The constant term in the Laurent series expansion.

b. The coefficient of the term with $(z-2)^{-1}$

c. The coefficient of the term with $(z-2)^{-3}$

d. The coefficient of the term with the highest positive power.

Hint: Consider the order of the pole and the corresponding term.

8. What is the residue of f(z) = $\frac{e^z}{z^2}$at z=0?

a. 1 b. 0

c. 1/2 d. -1

Hint: Identify the term with $(z-0)^{-1}$ in the Laurent series.

9. If f(z) has an essential singularity at z = 1, what can be said about the behavior of f(z) near z = 1?

a. f(z) has a removable singularity at z = 1.

b. f(z) is holomorphic near z = 1.

c. f(z) has an infinite number of poles near z = 1.

d. f(z) cannot be expanded in a Laurent series near z = 1

Hint: Think about the nature of essential singularities.

10. What is the residue of $\frac{1}{(z-1)^2(z+2)}$

a. -1/3 b. 1/3

c. ½ d. -1/4

Hint: Identify the term with $(z-1)^{-1}$ in the Laurent series.

11. What is the Cauchy Residue Theorem primarily used for in complex analysis?
   a. Integration of real-valued functions
   b. Integration of complex-valued functions
   c. Evaluation of limits
   d. Solving differential equations
   Hint: Think about the key property of the Cauchy Residue Theorem.

12. In the context of the Cauchy Residue Theorem, what is a residue?
   a. A singularity of a function
   b. A complex number associated with a singularity
   c. The derivative of a complex function
   d. The integral of a complex function
   Hint: Residues are related to singularities.

13. What is the residue at a simple pole of a function?
   a. Always zero
   b. The coefficient of the 1/z term in the Laurent series expansion
   c. The value of the function at the pole
   d. The integral of the function around the pole
   Hint: Consider the Laurent series expansion around a simple pole.

14. What is the residue at a pole of order n?
   a. The coefficient of the $\frac{1}{z^{n-1}}$ term in the Laurent series expansion
   b. The integral of the function around the pole
   c. The value of the function at the pole
   d. Always zero
   Hint: Think about the general form of the Laurent series for a pole of

order \(n\).

15. How is the residue at infinity defined?
a. As the limit of the function as z approaches infinity
b. As the coefficient of the1/z term in the Laurent series expansion at infinity
c. As the integral of the function along a closed contour at infinity
d. As the value of the function at infinity
Hint:Consider the behavior of the function as z goes to infinity.

16. What is the residue at infinity for a rational function?
a. Always zero
b. The sum of the residues at its finite poles
c. The value of the function at infinity
d. The integral of the function around the point at infinity
Hint: Think about the behavior of rational functions at infinity.

17. When does the residue at infinity play a significant role in the evaluation of integrals?
a. Only for functions with poles at infinity
b. Only for functions with essential singularities
c. Only for functions with simple poles
d. For functions with poles both at finite points and at infinity
Hint: Consider cases where the contour includes the point at infinity.

18. What is the relationship between the residue at infinity and the contour integral?
a. The residue at infinity is irrelevant to the contour integral
b. The contour integral is equal to the sum of residues at finite points

c. The contour integral is equal to 2πi times the residue at infinity
d. The contour integral is always zero
Hint: Recall the residue theorem and its application.

19. In the context of the residue at infinity, what is the winding number
of a contour?
a. The number of times the contour encircles the origin
b. The number of singularities enclosed by the contour
c. The number of times the contour encircles infinity
d. The order of a pole inside the contour
Hint: Consider the behavior of the contour around infinity.

20. When can the residue at infinity be non-zero?
a. Only when the function has an essential singularity at infinity
b. Only when the function has a pole at infinity
c. Only when the function is not analytic at infinity
d. Only when the function has a removable singularity at infinity
Hint: Think about the conditions for the residue at infinity to be non-zero.

21. What is the defining property of a removable singularity?**
a. The function is unbounded at the singularity.
b. The singularity can be removed by redefining the function at that point.
c. The singularity is an essential singularity.
d. The function is not defined at the singularity.
Hint: Consider the behavior of the function as it approaches the singularity.

22. Which of the following statements is true for a pole of order k in a complex function?

   a. The function is bounded at the pole.
   b. The function is not defined at the pole.
   c. The Laurent series expansion has a finite number of terms.
   d. The residue at the pole is zero.

   Hint: Recall the properties of poles and the Laurent series expansion

23. What is the characteristic feature of an essential singularity in a complex function?

   a. The singularity can be removed by redefining the function.
   b. The function is bounded at the singularity.
   c. The Laurent series expansion has infinitely many terms.
   d. The residue at the singularity is zero.

   Hint: Think about the behavior of the function near an essential singularity.

24. At which type of isolated singular point does the function approach infinity?**

   a. Removable singularity

   b. Pole
   c. Essential singularity
   d. None of the above

   Hint: Consider the behavior of the function as it approaches each type of singular point.

25. What is the relationship between the order of a pole and the growth of the function at that point?

a. Higher order poles imply slower growth of the function.

b. Higher order poles imply faster growth of the function.

c. The order of the pole does not affect the growth of the function.

d. Poles are always associated with bounded growth.

Hint: Consider how the order of a pole influences the terms in the Laurent series.

26. Which of the following is a common example of a removable singularity?**

a. $\frac{sin(z)}{z}$ b. $\frac{1}{e^z-1}$

c. $\frac{z}{e^z+1}$ d. $\frac{1}{cos(z)}$

Hint: Consider functions that can be extended analytically at the singularity.

27. At which type of isolated singular point does the Laurent series expansion include only negative powers of $(z-z_0)$?

a. Removable singularity

b. Pole

c. Essential singularity

d. None of the above

Hint: Think about the form of the Laurent series near different types of singular points.

28. What is the main characteristic of a removable singularity in

terms of
its residue?

a. The residue is always zero.
b. The residue is always finite.
c. The residue is infinite.
d. The concept of residue is not applicable for removable singularities.

Hint: Recall the definition and properties of residue at singular points.

29. For a function with an essential singularity, how does the Laurent series expansion behave?

a. It includes only positive powers of $(z-z_0)$
b. It includes both positive and negative powers of $(z-z_0)$.
c. It includes only negative powers of $(z-z_0)$.
d. The Laurent series expansion is not defined at essential singularities.

Hint:Consider the behavior of the Laurent series near an essential singularity.

30. Which of the following functions has an essential singularity at $z = 0$ ?

a. $e^z$ b. $\sin(\frac{1}{z})$
c. $\frac{1}{z^2+1}$ d. $\log(z)$

Hint: Focus on functions that exhibit non-algebraic behavior near the
origin.

31. What is the residue at a simple pole for the function f(z)= $\frac{2z}{z^2+1}$?

a. 1 b. 2

c. -2 d. 0

Hint: Recall the formula for the residue at a simple pole and the definition
of a simple pole.

32. For the function g(z)=$\frac{e^z}{z^2+1}$, what type of pole does it have at z = 2i ?

a. Simple pole b. Double pole

c. Triple pole d. No pole at = 2i

Hint: Examine the denominator and determine the order of the pole.

33. What is the residue at a pole of order 3 for the function ?

a. 0 b. 1

c. 2 d. -2

Hint: Use the formula for the residue at a pole of order k.

34. Consider the function k(z)= $\frac{z^2-1}{(z+i)^2}$ What is the residue at the double pole z = -i ?

a. 0 b. 1

c. 2i d. -2i

Hint:Determine the order of the pole and then calculate the residue.

35. For the function $\frac{sinz}{z^2-4}$, which type of singularity does it have at z = 2?

a. Removable singularity b. Simple pole

c. Essential singularity d. No singularity at z = 2

Hint: Examine the behavior of the function near z = 2 and identify the
type of singularity.

36. What is the residue at the pole z = -3 for the function f(z) $=\frac{2z^2}{(z+3)^2(z-1)}$

a. 2 b. 1

c. -2 d. -1

Hint:Identify the order of the pole and use the formula for the residue at a simple pole.

37. For the function g(z) = $\frac{e^{2z}}{(z-i)^3}$, what is the residue at the triple pole z = i ?

a. 0 b. 2/3

c. ½ d. 1/3

Hint: Recognize the order of the pole and apply the formula for the residue at a pole of order k.

38. Consider the function $\frac{cosz}{(z-\pi)^2}$. What is the residue at the double pole
z=π?

a. 0 b. -1

c. $\frac{1}{\pi}$ d. $\frac{\pi}{2}$

Hint:Determine the order of the pole and then compute the residue.

39. What type of singularity does the function k(z)= $\frac{e^z}{z^2+1}$ have at z = i ?

a. Removable singularity
b. Simple pole
c. Essential singularity
d. No singularity at z = i

Hint: Analyze the behavior of the function near z = i to identify the singularity type.

40. For the function m(z) = $\frac{\sin z}{z^3-1}$, what is the residue at the pole of order
3 at z = 1 ?

a. 0 b. 2/3
c. ½ d. 1/6

Hint: Determine the order of the pole and then use the formula for the residue at a pole of order k.

41. What is a zero of an analytic function?

a. A point where the function is not defined.
b. A point where the function takes the value zero.
c. A point where the function is unbounded.
d. A point where the function has a singularity.

Hint: Think about the behavior of the function at points where it equals zero.

42. If f(z) has a zero of order 2 at $z = 1$ , what can be said about f(z) near
$z = 1$ ?

a. f(z) has a removable singularity at $z = 1$.

b. f(z) has a pole of order 2 at $z = 1$ .

c. f(z) has an essential singularity at $z = 1$

d. f(z) is analytic a t$z = 1$.

Hint:Consider the behavior of f(z) near a zero of order $k$ .

43. Which of the following statements is true regarding the zeros of an
entire function?

a. Entire functions cannot have zeros.

b. Entire functions have isolated zeros.

c. Entire functions have zeros only at infinity.

d. Entire functions have an infinite number of zeros.

Hint: Recall the definition and properties of entire functions.

24. If $g(z) = (z - 2)(z + 3)^2$, how many zeros does g(z) have in the complex plane?

a. 1 b. 2

c. 3 d. 4

Hint: Count the number of distinct factors in the expression for g(z) .

45. Consider the function $h(z) = e^z - 1$ What can be said about the zeros of h(z) ?

a. h(z) has no zeros.
b. h(z) has a simple zero at $z = 0$ .
c. h(z) has an essential singularity at $z = 0$ .
d. h(z) has a pole at $z = 0$ .
Hint:Examine the behavior of h(z) at $z = 0$ .

46. What is the order of a zero of an analytic function at a point?
a. The value of the function at that point.
b. The multiplicity of the zero at that point.
c. The residue of the function at that point.
d. The reciprocal of the derivative of the function at that point.
Hint:Consider how the function is factored near a zero.

47. If f(z) has a pole of order 3 at $z = 2$ , what is the behavior of f(z) near
$z = 2$ ?
a. f(z) is bounded at $z = 2$ .
b. f(z) has an essential singularity at $z = 2$ .
c. f(z) approaches infinity as z approaches 2.

d. f(z) is not defined at $z = 2$ .
Hint: Consider the definition and properties of poles.

48. For an entire function, which statement is true regarding its zeros?
a. Entire functions have only real zeros.
b. Entire functions have only simple zeros.
c. Entire functions have an isolated number of zeros.
d. Entire functions may have an infinite number of zeros.

Hint: Recall the characteristics of entire functions.

49. If g(z) = z(z-1)(z+1) , what can be said about the zeros of g(z)?
   a. g(z) has a triple zero at z = 0 .
   b. g(z) has simple zeros at z = 0, 1,and -1 .
   c. g(z) has a double zero at z = 0 .
   d. g(z) has a pole at z = 0 .
   Hint: Examine the factors in the expression for g(z) .

50. Consider the function h(z) = $\frac{1}{z^2+1}$. What is the type of singularity at z = i ?
   a. Removable singularity
   b. Simple pole
   c. Essential singularity
   d. No singularity at z = i
   Hint: Examine the behavior of the function near z = i .

**ANSWERS**

1., 2., 3.c, 4.c, 5.c, 6.b, 7.c, 8.a, 9.d, 10.b, 11.b, 12.b, 13.b, 14.a, 15.b, 16.a, 17.d, 18.c, 19.c, 20.a, 21.b, 22.c, 23.c, 24.b, 25.b, 26.b, 27.b, 28.a, 29.b, 30.b, 31.a, 32.b, 33.b, 34.d, 35.b, 36.a, 37.d, 38.c, 39.b, 40.d, 41.b, 42.d, 43.d, 44.c, 45.b, 46.b, 47.c, 48.d, 49.b, 50.b.

**UNIT-V**

1. What is the image of the point z = i under the transformation w = 1/z

a. w = -i b. w = i

c. w = -1 d. w = 1

Hint:Substitute z = i into the transformation and simplify.

2. Under the transformation w = 1/z, what happens to the origin z = 0 ?

a. The origin is mapped to w =∞

b. The origin is mapped to $w = 0$
c. The origin is mapped to $w = 1$ .
d. The origin is not defined under the transformation.
Hint: Consider the behavior of the transformation at $z = 0$

3. If $z = 2 + 3i$ , what is the real part of the corresponding w under the transformation $w = 1/z$
a. 2/13
b. 3/13
c.13/2
d. 13/13
Hint: Substitute $z = 2 + 3i$ into the transformation, then find the real part of $w$ .

4. What is the image of the circle $|z - 1| = 2$ under the transformation $w = 1/z$

a. $|w - 1| = 2$
b. $|w + 1| = 2$
c. $|w| = ½$
d. $|w| = 2$

5. Under the transformation w=1/z, what happens to the imaginary axis (Im(z)=1)?
a. The imaginary axis is mapped to the axis
b. The imaginary axis is mapped to the imaginary axis.
c. The imaginary axis is not defined under the transformation.
d. The imaginary axis is mapped to the real axis.

6. What is the image of the real axis under the mapping \( w = \frac{1}{z} \)?**

a. The imaginary axis
b. The unit circle
c. The entire complex plane
d. The real axis itself

Hint: Substitute z = x into the mapping and observe the resulting expression.

7. Under the linear fractional transformation w = $\frac{2z-1}{z+3}$, what is the image of the point z = -3

a. w = -1 b. w = -2
c. w = 1 d. w = 2

Hint: Substitute z = -3 into the transformation and simplify.

8. For the linear fractional transformation $\frac{1}{z-2}$, where does the pole occur?**

a. z = 1 b. z = 2
c. z = 0 d. z = -2

Hint: Identify the value of z that makes the denominator zero.

9. Consider the linear fractional transformation w = $\frac{z-i}{z+i}$. What is the image of the imaginary axis under this transformation?

a. The real axis
b. The unit circle
c. The vertical line Re(w)=0
d. The horizontal line Im(z)=0

Hint: Substitute z = iy into the transformation and simplify.

10. If w= $\frac{1}{z}$ what is the image of the circle $|z - i| = 2$ ?

a. $|w - i| = \frac{1}{2}$ b. $|w + i| = 2$

c. $|w| = 2$ d. $|w + i| = \frac{1}{2}$

Hint: Substitute $z = i + 2e^{i\theta}$ into the transformation and simplify.

11. What is the implicit form of a linear fractional transformation?

a. az + b b. $\frac{az+b}{cz+d} = 1$

c. $\frac{az+b}{cz+d}$ d. $\frac{a}{z}$ +b

Hint: Recall the general form of a linear fractional transformation.

12. For a linear fractional transformation $\frac{2z-1}{z+3}$, what is the implicit form?

a. 2z - 1 = z + 3 b. 2w - 1 = z + 3

c. 2z - 1 = w + 3 d. w + 3 = 2z - 1

Hint: Cross-multiply to obtain an equation involving Z & W

13. Which of the following is an implicit equation for the linear fractional transformation w=$\frac{1}{z-2}$

a. w - 2 = z b. w(z - 2) = 1

c. wz - 2w = 1 d. wz = 2w + 1

Hint:Cross-multiply and simplify to get an implicit equation.

14. Consider the linear fractional transformation w =$\frac{z-i}{z+i}$. What is the implicit equation?**

a. (z + i)w = z - i b. (z - i)w = z + i

c. $(z + i)w = z + i$ d. $(z - i)w = z - i$

Hint: Cross-multiply and simplify.

15. If w=1/z, what is the implicit equation relating w and z?

a. $wz = 1$

b. $w + z = 1$

c. $wz = 0$

d. $w - z = 1$

Hint:Cross-multiply and simplify.

16. What is the essential property of conformal mappings in terms of angles?

a. They always double angles.

b. They always preserve angles.

c. They always reverse angles.

d. They have no effect on angles.

Hint:Think about the geometric property that conformal mappings preserve.

17. In a conformal mapping, what happens to the angle between two curves at the point of mapping?

a. It is halved. b. It is doubled.

c. It is preserved. d. It becomes zero.

Hint:Consider the behavior of angles under conformal mappings.

18. For a conformal mapping $w = e^z$, what can be said about the angles between curves in the z-plane and the w-plane?

a. Angles are halved.

b. Angles are doubled.
c. Angles are preserved.
d. Angles become zero.
Hint:Consider the geometric effect of the exponential function on angles.

19. What type of transformation is always conformal in the complex plane?
a. Translation
b. Scaling
c. Rotation
d. Linear fractional transformation
Hint:Recall the properties of conformal transformations.

20. In a conformal mapping, what happens to the cross-ratio of four points in the \(z\)-plane and their images in the w-plane?
a. It is halved. b. It is doubled.
c. It is preserved. d. It becomes zero.
Hint: Understand the behavior of cross-ratio under conformal mappings.

21. What is the scale factor of a complex function at a point?
a. The derivative of the function at that point.
b. The reciprocal of the derivative of the function at that point.
c. The real part of the function at that point.
d. The imaginary part of the function at that point.
Hint: Consider the relationship between the function and its

derivative.

22. For a complex functionf(z) = z^2 , what is the scale factor at the point z = 2 ?

a. 2 b. 4

c. 8 d. 16

Hint: Find the derivative of f(z) with respect to z and evaluate it at z = 2 .

23. What is the local inverse of a complex function at a point?

a. The reciprocal of the function.

b. The derivative of the function.

c. A function that undoes the effects of the original function locally.

d. A function that maps all points to the origin.

Hint: Think about what it means for a function to have a local inverse.

24. For a complex function g(z) = e^z , what is the scale factor at the point z = 0 ?

a. 0 b. 1

c. e d. e^2

Hint: Find the derivative of g(z) with respect to z and evaluate it at z = 0.

25. What condition is necessary for a complex function to have a local inverse at a point?

a. The function must be continuous at that point.

b. The function must be differentiable at that point.

c. The function must be bijective in a neighborhood of that point.
d. The function must be periodic.

Hint: Consider the properties that a function must have to admit a local inverse.

26. What is the scale factor of a complex function f(z) = z^3at the origin z = 0?

a. 0 b. 1
c. 3 d. 9

Hint: Find the derivative of f(z) with respect to z and evaluate it at z = 0.

27. If a complex function h(z) has a scale factor of 2 at a point z = a, what is the scale factor of h(z - a) at z = 0?

a. 0 b. 1
c. 2 d. 4

Hint: Consider the effect of translation on the scale factor.

28. What is the local inverse of the complex function g(z) = sin(z) at the point z = $\frac{\pi}{2}$?

a. $\sin^{-1}(z)$ b. cos(z)
c. tan(z) d. sec(z)

Hint: Think about which trigonometric function is the inverse of sin(z).

29. For a complex function f(z) = e^{2z}, what is the scale factor at any point z?

a. 0 b. 1

c. 2 d. e^2

Hint: Find the derivative of f(z)with respect to z at any point z.

30. Under what condition does a complex function have a local uinverse at every point in its domain?

a. The function must be differentiable everywhere.

b. The function must be continuous everywhere.

c. The function must be periodic.

d. The function must be bijective everywhere.

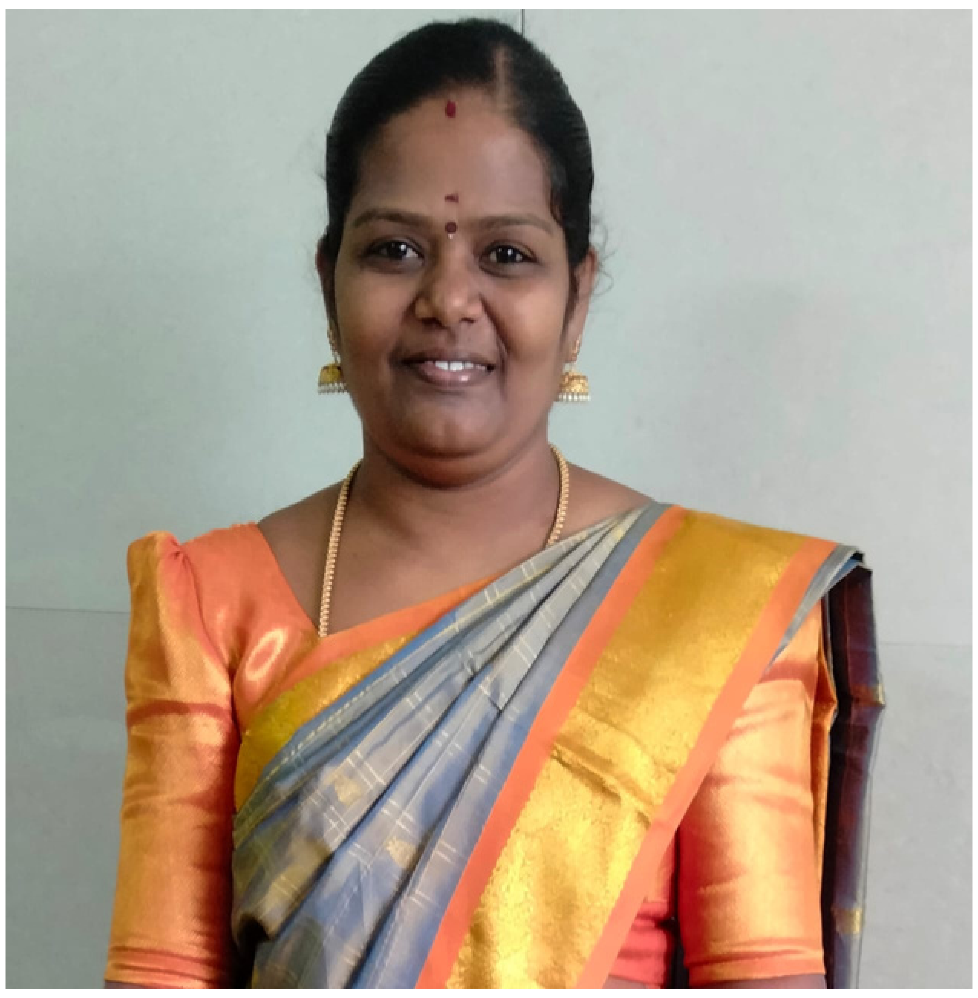
Hint:Consider the criteria for a function to have a local inverse.

**ANSWERS**

1.a, 2.a, 3.a, 4.b, 5.a, 6.a, 7.a, 8.a, 9.c, 10.b, 11.b, 12.c, 13.c, 14.a, 15.a, 16.b, 17.c, 18.c, 19.c, 20.c, 21.b, 22.b, 23.c, 24.c, 25.a, 26.a, 27.c, 28.a, 29.c, 30.d.

**ABOUT THE AUTHOR**

Mrs. M. Meenakshi was born in 1984 at Coimbatore. Completed her school in Vellakovil and College in Erode. Did UG & PG in Maharaja College for Women, Perundurai; M.Phil in Sri Vasavi College in Cithode and currently doing Ph.D in Sree GVG Vishalakshi College in Udumalpet. Worked as an Assistant Professor at Tirupur Kumaran College For Women at Tirupur from 2009 till 2017. From 2017 to till date working as an Assistant Professor in the Department of Mathematics at St.Joseph's College of Arts and Science for Women, Hosur. Published 10 National & International Conference Proceedings. Area of interest in research is Graph Theory.



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