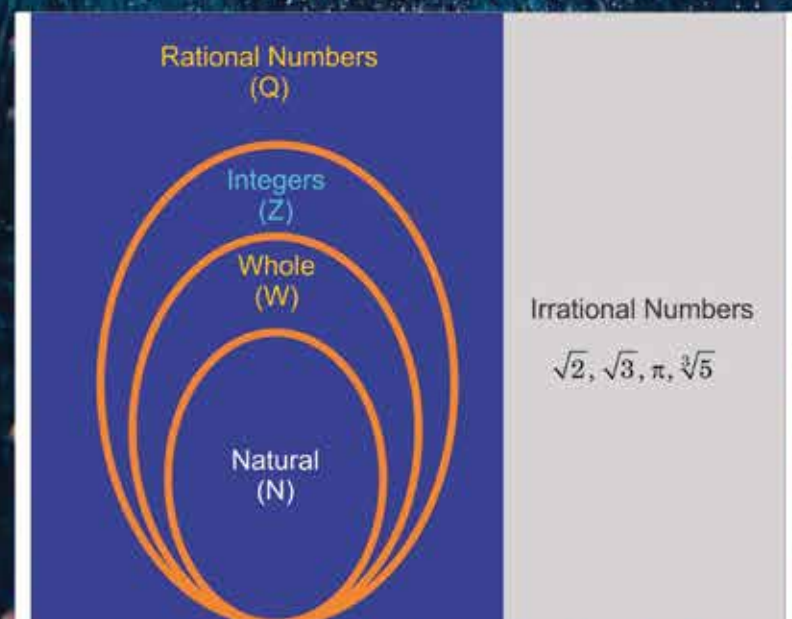


MCCQ ON NUMBER THEORY



MUQ ON NUMBER THEORY

UNIT-I:

Divisibility Introduction- Divisibility, Greatest Common Divisor, Euclid's Algorithm, Greatest Common Divisor via Euclid's Algorithm- Least Common Multiple Representation of Integers, Decimal Representation of Integers, Binary Representation of Integers(Chapter:2. Sections 2.1 to 2.4, Related Problems)

UNIT-II:

Primes Introduction-Primes, Prime counting function, prime number theorem, Test of primality by trial division – Sieve of Eratosthenes, Canonical Factorization, Fundamental theorem of arithmetic, Sieve of Eratosthenes, Determining the canonical factorization of a natural number (Chapter3: Sections-3.1 to 3.3, Related Problems)

UNIT-III:

Congruences Introduction-Congruences and Equivalence Relations, Equivalence Relations and Linear Congruences - Linear Diophantine Equations and the Chinese Remainder Theorem (Chapter4: Sections 4.1 to 4.4, Related Problems)

UNIT-IV:

Congruences(continued) Polynomial Congruences- Modular Arithmetic: Fermat's theorem – Wilson's Theorem and Fermat's Numbers – Pythagorean Equation(Chapter4: Sections 4.5 to 4.8, Related Problems)

UNIT-V:

Arithmetic Functions Introduction- Sigma function, Tau function, Dirichlet product – Dirichlet Inverse, Moebius function, Euler's function, Euler's Theorem, An application of algebra (Chapter5: Sections 5.1 to 5.3, Related Problems)

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1. The Division algorithm:

1. What is the division algorithm in number theory?

- a. Algorithm for long division
- b. Algorithm for finding prime numbers
- c. Algorithm for dividing two integers
- d. Algorithm for multiplication

2. Which of the following statements best describes the division algorithm?

- a. It states that any integer divided by zero is undefined.
- b. It states that any integer divided by another integer will yield a quotient and a remainder.
- c. It states that any two integers have a common divisor.
- d. It states that any integer can be divided by any other integer without a remainder.

3. The division algorithm is often used to find:

- a. Prime numbers
- b. GCD (Greatest Common Divisor)
- c. LCM (Least Common Multiple)
- d. Square roots

4. In the division algorithm, if a and b are integers, where $a > b$, what is the quotient (q) and remainder (r)?

- a. $q = a/b$, $r = a \% b$
- b. $q = a \% b$, $r = a/b$
- c. $q = a/b$, $r = b \% a$
- d. $q = a/b$, $r = a - b*q$

5. Which of the following is a correct representation of the division algorithm?

a. $a = bq + r$

b. $a = bq - r$

c. $a = b/q + r$

d. $a = b - r/q$

Answers:

1. c

2. b

3. b

4. d

5. a

Hints:

1. Think about the purpose of the division algorithm in number theory.
2. Consider what the division algorithm provides when one integer is divided by another.
3. What are some common applications of the division algorithm in number theory?
4. Focus on the relationship between the dividend, divisor, quotient, and remainder in the division algorithm.
5. Look for the correct representation of the division algorithm with the correct placement of variables.

1. What is the division algorithm in number theory used for?

- a) Division of polynomials
- b) Division of integers
- c) Division of complex numbers

d) Division of matrices

Answer: b) Division of integers

2. In the division algorithm, if 'a' is divided by 'b', what are the quotient 'q' and remainder 'r' such that $a = bq + r$?

a) $q = a/b$, $r = a \% b$

b) $q = b/a$, $r = a \% b$

c) $q = a/b$, $r = b \% a$

d) $q = b/a$, $r = b \% a$

Answer: a) $q = a/b$, $r = a \% b$

3. What is the condition for the division algorithm to hold true for integers 'a' and 'b'?

a) $a \geq b$

b) $a > b$

c) $a \neq 0$

d) $b \neq 0$

Answer: d) $b \neq 0$

4. If $a = 23$ and $b = 5$, what is the quotient 'q' in the division algorithm?

a) 4

b) 5

c) 6

d) 7

Answer: c) 6

5. What is the remainder 'r' in the division algorithm for $(a = 89)$ and $(b = 7)$?

- a) 3
- b) 5
- c) 6
- d) 8

Answer: b) 5

6. If $(a = 50)$ and $(b = 7)$, what is the remainder 'r' in the division algorithm?

- a) 1
- b) 2
- c) 3
- d) 4

Answer: d) 4

7. Which of the following statements is true regarding the division algorithm?

- a) The remainder is always greater than the divisor.
- b) The remainder is always less than the divisor.
- c) The remainder is always equal to the divisor.
- d) The remainder is always non-negative and less than the divisor.

Answer: d) The remainder is always non-negative and less than the divisor.

8. What is the division algorithm often used for in elementary number theory?

- a) Solving linear equations
- b) Prime factorization
- c) Finding greatest common divisors
- d) Calculating square roots

Answer: c) Finding greatest common divisors

9. If $(a = 105)$ and $(b = 8)$, what is the quotient 'q' in the division algorithm?

- a) 11
- b) 12
- c) 13
- d) 14

Answer: b) 12

10. In the division algorithm, what is the role of the remainder 'r'?

- a) It is the result of the division.
- b) It is ignored in the algorithm.
- c) It is the leftover after the division.
- d) It is always zero.

Answer: c) It is the leftover after the division.

11. If $(a = 72)$ and $(b = 9)$, what is the remainder 'r' in the division algorithm?

- a) 5

- b) 6
- c) 7
- d) 8

Answer: b) 6

12. What is the division algorithm's primary application in cryptography?

- a) Key generation
- b) Data encryption
- c) Prime number generation
- d) Hash function design

Answer: a) Key generation

13. If $(a = 123)$ and $(b = 11)$, what is the quotient 'q' in the division algorithm?

- a) 10
- b) 11
- c) 12
- d) 13

Answer: c) 12

14. In the division algorithm, what is the significance of the divisor 'b'?

- a) It is the result of the division.
- b) It is the remainder.
- c) It is the number being divided.

d) It is always zero.

Answer: c) It is the number being divided.

15. If $(a = 65)$ and $(b = 8)$, what is the remainder 'r' in the division algorithm?

- a) 1
- b) 2
- c) 3
- d) 4

Answer: a) 1

16. Which of the following is a necessary condition for the division algorithm to be valid for integers 'a' and 'b'?

- a) $(a > b)$
- b) $(a = 0)$
- c) $(b = 0)$
- d) $(a \geq 0)$

Answer: c) $(b = 0)$

17. If $(a = 98)$ and $(b = 7)$, what is the quotient 'q' in the division algorithm?

- a) 12
- b) 13
- c) 14
- d) 15

Answer: b) 13

18. What is the division algorithm's role in modular arithmetic?

- a) It is used to compute modular inverses.
- b) It is used to find modular residues.
- c) It is not applicable in modular arithmetic.
- d) It is used to perform modular exponentiation.

Answer: b) It is used to find modular residues.

19. If $(a = 35)$ and $(b = 4)$, what is the remainder 'r' in the division algorithm?

- a) 1
- b) 2
- c) 3
- d) 4

Answer: c) 3

20. What is the division algorithm's role in the Euclidean algorithm for finding the greatest common divisor (GCD)?

- a) It is not used in the Euclidean algorithm.
- b) It helps to compute quotients and remainders.
- c) It is used to generate prime numbers.
- d) It is used for modular exponentiation.

Answer: b) It helps to compute quotients and remainders.

21. If $(a = 124)$ and $(b = 9)$, what is the quotient 'q' in the division algorithm?

- a) 13
- b) 14
- c) 15
- d) 16

Answer: b) 14

Question 1: What is the division algorithm in number theory used for?

- a) Division of polynomials
- b) Division of integers
- c) Division of complex numbers
- d) Division of fractions

Answer: b) Division of integers

Question 2: In the division algorithm, if a and b are integers, with b not equal to zero, what does the algorithm guarantee?

- a) Division with remainder
- b) Division without remainder
- c) Multiplication with remainder
- d) Addition with remainder

Answer: a) Division with remainder

Question 3: What is the result of the division algorithm expressed as a formula?

- a) $a = bq + r$

b) $a = b/q + r$

c) $a = b - r$

d) $a = bq - r$

Answer: a) $a = bq + r$

Question 4: In the formula $a = bq + r$, what does 'a' represent?

a) Quotient

b) Divisor

c) Dividend

d) Remainder

Answer: c) Dividend

Question 5: If you have 23 divided by 5 using the division algorithm, what is the quotient and remainder?

a) Quotient: 5, Remainder: 3

b) Quotient: 4, Remainder: 3

c) Quotient: 4, Remainder: 5

d) Quotient: 5, Remainder: 4

Answer: b) Quotient: 4, Remainder: 3

Question 6: What happens if the divisor in the division algorithm is zero?

a) Undefined

b) Quotient is zero

c) Remainder is zero

d) No effect on the algorithm

Answer: a) Undefined

Question 7: How is the remainder 'r' determined in the division algorithm?

- a) It is the product of a and b
- b) It is the difference between a and bq
- c) It is the sum of a and b
- d) It is the product of q and b

Answer: b) It is the difference between a and bq

Question 8: Which of the following statements is true about the division algorithm?

- a) The remainder is always greater than the divisor
- b) The remainder is always less than the divisor
- c) The remainder is always less than or equal to the divisor
- d) The remainder is always greater than or equal to the divisor

Answer: c) The remainder is always less than or equal to the divisor

Question 9: What is the division algorithm used for in computer science?

- a) Sorting algorithms
- b) Arithmetic calculations
- c) File compression
- d) Encryption

Answer: b) Arithmetic calculations

Question 10: If $a = 17$ and $b = 3$, what is the quotient and remainder in the division algorithm?

- a) Quotient: 6, Remainder: 2
- b) Quotient: 5, Remainder: 2
- c) Quotient: 6, Remainder: 1
- d) Quotient: 5, Remainder: 1

Answer: a) Quotient: 6, Remainder: 2

Question 11: In the division algorithm, what is the condition for the remainder 'r' to be zero?

- a) $a = bq$
- b) $a > bq$
- c) $a < bq$
- d) $a = bq + r$

Answer: a) $a = bq$

Question 12: Which of the following is a correct statement about the division algorithm?

- a) The quotient is always positive
- b) The remainder is always negative
- c) The quotient is always non-negative
- d) The remainder is always positive

Answer: c) The quotient is always non-negative

Question 13: If $a = -25$ and $b = 7$, what is the quotient and remainder in the division algorithm?

- a) Quotient: -3, Remainder: 4
- b) Quotient: -4, Remainder: 6
- c) Quotient: -4, Remainder: 3
- d) Quotient: -3, Remainder: 6

Answer: c) Quotient: -4, Remainder: 3

Question 14: What is the relationship between the divisor, quotient, and remainder in the division algorithm?

- a) Divisor = Quotient * Remainder
- b) Quotient = Divisor * Remainder
- c) Dividend = Quotient * Divisor + Remainder
- d) Quotient = Dividend / Divisor - Remainder

Answer: c) Dividend = Quotient * Divisor + Remainder

Question 15: How is the quotient 'q' determined in the division algorithm?

- a) It is the product of a and b
- b) It is the sum of a and r
- c) It is the difference between a and r
- d) It is the result of a divided by b

Answer: d) It is the result of a divided by b

Question 16: In the division algorithm, what is the significance of the condition $0 \leq r < |b|$?

- a) It ensures the remainder is positive
- b) It ensures the remainder is non-negative
- c) It ensures the remainder is less than the divisor
- d) It ensures the remainder is less than the absolute value of the divisor

Answer: d) It ensures the remainder is less than the absolute value of the divisor

Question 17: If $a = 50$ and $b = -8$, what is the quotient and remainder in the division algorithm?

- a) Quotient: -6, Remainder: 2
- b) Quotient: -7, Remainder: 6
- c) Quotient: -7, Remainder: 2
- d) Quotient: -6, Remainder: 6

Answer: c) Quotient: -7, Remainder: 2

Question 18: Which of the following is a correct representation of the division algorithm?

- a) $a = bq - r$
- b) $a = b - rq$
- c) $a = r - bq$
- d) $a = bq + r$

Answer: d) $a = bq + r$

Question 19: If $a = 18$ and $b = 5$, what is the quotient and remainder in the division algorithm?

- a) Quotient: 4, Remainder: 2

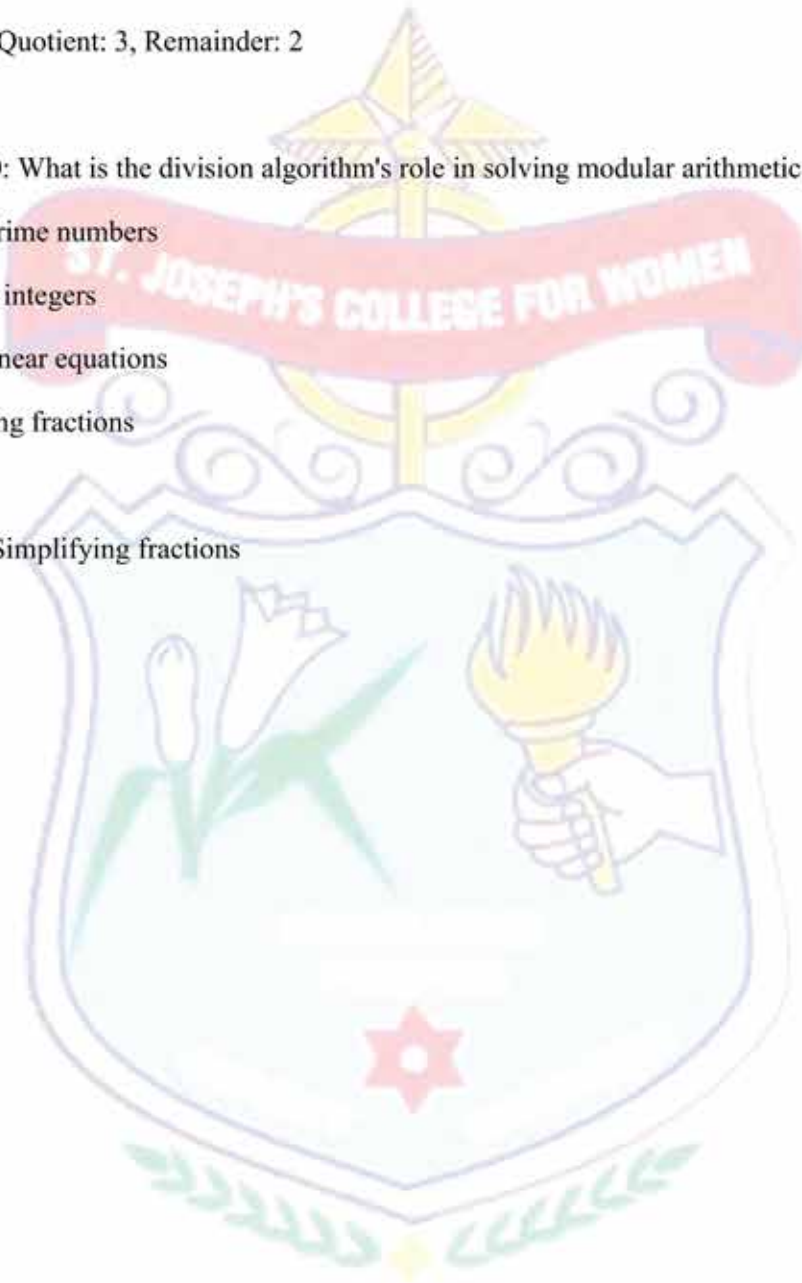
- b) Quotient: 3, Remainder: 3
- c) Quotient: 3, Remainder: 2
- d) Quotient: 4, Remainder: 3

Answer: c) Quotient: 3, Remainder: 2

Question 20: What is the division algorithm's role in solving modular arithmetic problems?

- a) Finding prime numbers
- b) Factoring integers
- c) Solving linear equations
- d) Simplifying fractions

Answer d) Simplifying fractions



Prime and composite number:

Question 1: What is a prime number?

- a) A number divisible by 1 and itself only
- b) A number divisible by 2 and itself only
- c) A number divisible by 1, 2, and itself
- d) A number divisible by 1, 2, and 3 only

Answer: b) A number divisible by 2 and itself only

Question 2: How many prime numbers are there between 1 and 10?

- a) 2
- b) 3
- c) 4
- d) 5

Answer: d) 5 (2, 3, 5, 7)

Question 3: What is the smallest prime number?

- a) 0
- b) 1
- c) 2
- d) 3

Answer: c) 2

Question 4: Which of the following is a composite number?

- a) 2
- b) 3
- c) 4
- d) 5

Answer: c) 4

Question 5: What is the only even prime number?

- a) 2
- b) 4
- c) 6
- d) 8

Answer: a) 2

Question 6: What is the sum of the first two prime numbers?

- a) 3
- b) 5
- c) 7
- d) 9

Answer: b) 5 (2 + 3)

Question 7: If a number is not prime, what is it called?

- a) Odd number
- b) Composite number

- c) Natural number
- d) Rational number

Answer: b) Composite number

Question 8: What is the only positive integer that is neither prime nor composite?

- a) 0
- b) 1
- c) 2
- d) 3

Answer: b) 1

Question 9: If a number is divisible by 2 and 3, but not by 5, what can you conclude about it?

- a) It is prime
- b) It is composite
- c) It is odd
- d) It is a perfect square

Answer: b) It is composite

Question 10: What is the largest prime number less than 20?

- a) 15
- b) 17
- c) 19
- d) 21

Answer: c) 19

Question 11: Which of the following statements is true about prime numbers?

- a) Every prime number is odd
- b) Every prime number is even
- c) Every prime number is positive
- d) Every prime number is negative

Answer: a) Every prime number is odd

Question 12: If a number is divisible by 5 and 7, but not by 3, what can you conclude about it?

- a) It is prime
- b) It is composite
- c) It is odd
- d) It is a perfect square

Answer: a) It is prime

Question 13: What is the product of the first three prime numbers?

- a) 2
- b) 6
- c) 30
- d) 210

Answer: c) 30 ($2 * 3 * 5$)

Question 14: Which of the following numbers is neither prime nor composite?

- a) 0
- b) 1
- c) 2
- d) 3

Answer: b) 1

Question 15: How many factors does a prime number have?

- a) 0
- b) 1
- c) 2
- d) 3

Answer: c) 2 (1 and the number itself)

Question 16: If a number is divisible by 2, 5, and 7, what can you conclude about it?

- a) It is prime
- b) It is composite
- c) It is odd
- d) It is a perfect square

Answer: b) It is composite

Question 17: What is the only even prime number?

- a) 2
- b) 4
- c) 6
- d) 8

Answer: a) 2

Question 18: If a number is not divisible by any number other than 1 and itself, what is it?

- a) Prime
- b) Composite
- c) Odd
- d) Square number

Answer: a) Prime

Question 19: What is the sum of the first four prime numbers?

- a) 12
- b) 18
- c) 22
- d) 26

Answer: c) 22 ($2 + 3 + 5 + 7$)

Question 20: Which of the following is a prime number?

- a) 15
- b) 21

c) 29

d) 35

Answer: c) 29

Question 21: If a number is divisible by 3 and 11, but not by 7, what can you conclude about it?

a) It is prime

b) It is composite

c) It is odd

d) It is a perfect square

Answer: b) It is composite

Question 22: What is the smallest composite number?

a) 0

b) 1

c) 2

d) 4

Answer: d) 4

Question 23: How many prime numbers are there between 20 and 30?

a) 1

b) 2

c) 3

d) 4

Answer: b) 2 (23 and 29)

Question 24: What is the only prime number that is even?

- a) 2
- b) 3
- c) 5
- d) 7

Answer: a) 2

Question 25: If a number is not divisible by 2, 3, 5, or 7, what can you conclude about it?

- a) It is prime
- b) It is composite
- c) It is odd
- d) It is a perfect square

Answer: a) It is prime

Question 26: What is the only positive integer that is neither prime nor composite?

- a) 0
- b) 1
- c) 2
- d) 3

Answer: b) 1

Question 27: Which of the following is a composite number?

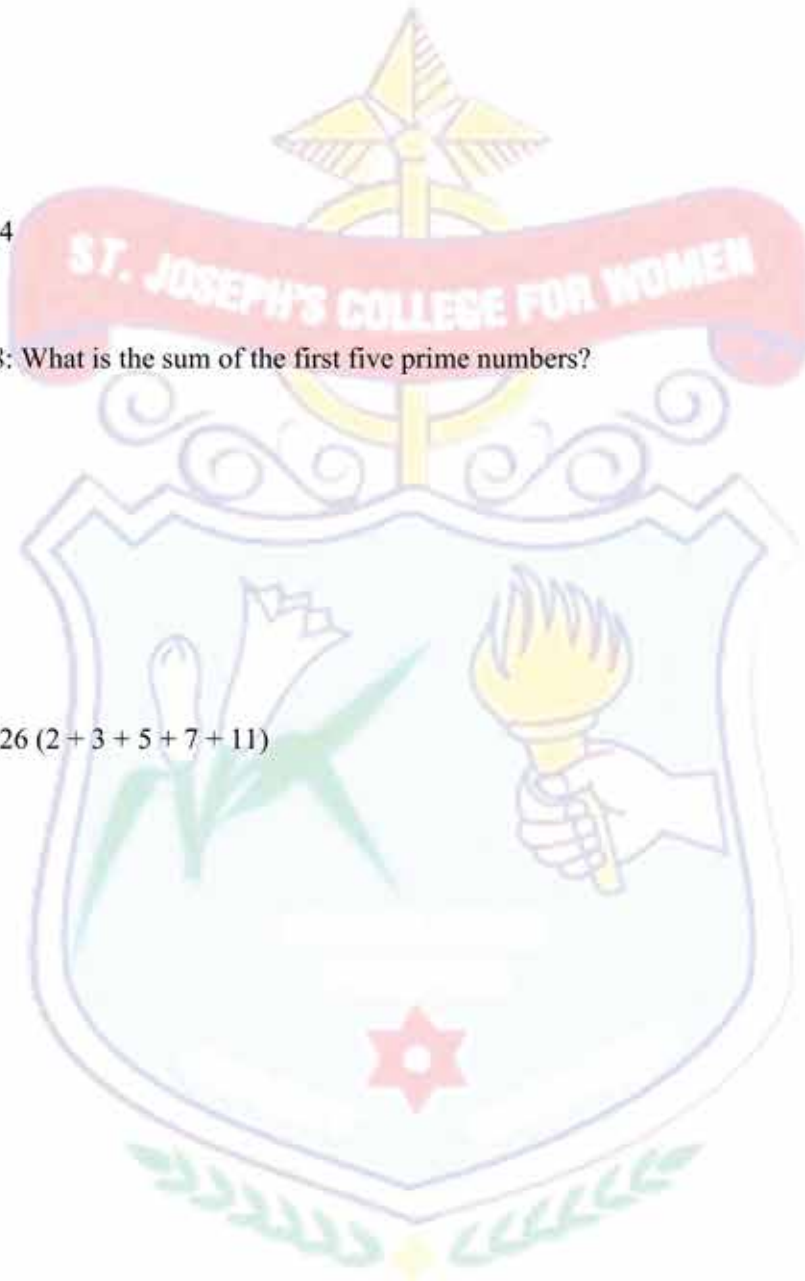
- a) 2
- b) 3
- c) 4
- d) 5

Answer: c) 4

Question 28: What is the sum of the first five prime numbers?

- a) 15
- b) 20
- c) 26
- d) 28

Answer: c) 26 ($2 + 3 + 5 + 7 + 11$)



Fibonacci and Lucas numbers

Question 1:

What is the next number in the Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, __?

- a) 10
- b) 11
- c) 13
- d) 21

Hint 1: Add the last two numbers in the sequence.

Answer: c) 13

Question 2:

In the Lucas sequence, what is the sum of the first two numbers: 2, 1, 3, 4, 7, __?

- a) 10
- b) 11
- c) 12
- d) 14

Hint 2: The Lucas sequence starts with 2 and 1.

Answer: a) 10

Question 3:

What is the 8th Fibonacci number?

- a) 13
- b) 21
- c) 34

d) 55

Hint 3: Use the formula $F(n) = F(n-1) + F(n-2)$.

Answer: c) 34

Question 4:

If $L(5)$ is the 5th Lucas number, what is its value?

- a) 3
- b) 7
- c) 11
- d) 18

Hint 4: Use the Lucas sequence formula.

Answer: b) 7

Question 5:

Which sequence is formed by adding corresponding Fibonacci and Lucas numbers: 3, 5, 8, 11, ___?

- a) Fibonacci sequence
- b) Lucas sequence
- c) Sum sequence
- d) Prime sequence

Hint 5: Add the numbers from both sequences.

Answer: c) Sum sequence

1. What is the first number in the Fibonacci sequence?

- a) 0
- b) 1
- c) 2
- d) 3

Hint: The Fibonacci sequence starts with 0 and 1.

Answer: b) 1

2. In the Fibonacci sequence, what is the sum of the first two numbers to get the third?

- a) 0
- b) 1
- c) 2
- d) 3

Hint: Each number in the Fibonacci sequence is the sum of the two preceding ones.

Answer: c) 2

3. What is the next number in the Fibonacci sequence after 13?

- a) 15
- b) 18
- c) 21

d) 24

Hint: Add the last two numbers to get the next one.

Answer: a) 15

4. Which of the following is not a Fibonacci number?

- a) 5
- b) 8
- c) 13
- d) 15

Hint: Check which number doesn't follow the Fibonacci sequence.

Answer: d) 15

5. What is the relationship between consecutive Fibonacci numbers as they go to infinity?

- a) They converge to a constant.
- b) They alternate between odd and even.
- c) They increase exponentially.
- d) They approach the golden ratio.

Hint: Consider the limit of the ratio between consecutive Fibonacci numbers.

Answer: d) They approach the golden ratio.

6. What is the Lucas number following 4 in the sequence?

- a) 7
- b) 8
- c) 9
- d) 10

Hint: The Lucas sequence follows the same rule as the Fibonacci sequence.

Answer: a) 7

7. In the Lucas sequence, what is the sum of the first two numbers to get the third?

- a) 1
- b) 2
- c) 3
- d) 4

Hint: Like the Fibonacci sequence, the Lucas sequence follows a similar rule.

Answer: b) 2

8. Which of the following is a property unique to the Lucas sequence and not the Fibonacci sequence?

- a) Exponential growth
- b) Alternation between odd and even
- c) Convergence to a constant
- d) Relationship to the golden ratio

Hint: Look for distinctive characteristics of the Lucas sequence.

Answer: b) Alternation between odd and even

9. What is the sum of the first five Fibonacci numbers?

- a) 8
- b) 12
- c) 20
- d) 40

Hint: Add the first five numbers in the Fibonacci sequence.

Answer: c) 20

10. Which Fibonacci number is also a triangular number?

- a) 3
- b) 5
- c) 8
- d) 13

Hint: Triangular numbers can be expressed as the sum of consecutive natural numbers.

Answer: b) 5

11. What is the relationship between the Fibonacci and Lucas sequences?

- a) They are completely unrelated.

- b) Lucas numbers are double the Fibonacci numbers.
- c) Lucas numbers are the squares of Fibonacci numbers.
- d) Lucas numbers are the sums of consecutive Fibonacci numbers.

Hint: Examine the relationship between the two sequences.

Answer: c) Lucas numbers are the squares of Fibonacci numbers.

12. What is the 10th Fibonacci number?

- a) 34
- b) 55
- c) 89
- d) 144

Hint: Count through the Fibonacci sequence to find the 10th number.

Answer: b) 55

13. In the Lucas sequence, what is the product of the first two numbers to get the third?

- a) 2
- b) 3
- c) 4
- d) 5

Hint: Unlike the sum in Fibonacci, Lucas sequence uses a different operation to generate the next number.

Answer: a) 2

14. What is the common ratio between consecutive Lucas numbers as they go to infinity?

- a) It approaches the golden ratio.
- b) It converges to a constant.
- c) It alternates between odd and even.
- d) It decreases exponentially.

Hint: Similar to the Fibonacci sequence, examine the limit of the ratio between consecutive Lucas numbers.

Answer: a) It approaches the golden ratio.

15. Which Lucas number follows 11 in the sequence?

- a) 15
- b) 18
- c) 22
- d) 26

Hint: Use the same rule as the Fibonacci sequence to find the next number in the Lucas sequence.

Answer: c) 22

16. What is the sum of the first eight Fibonacci numbers?

- a) 33
- b) 55

- c) 89
- d) 144

Hint: Add the first eight numbers in the Fibonacci sequence.

Answer: c) 89

17. Which of the following statements is true about both Fibonacci and Lucas sequences?

- a) They both start with 1.
- b) They both have alternating odd and even numbers.
- c) They both converge to the golden ratio.
- d) They both increase exponentially.

Hint: Look for similarities between the two sequences.

Answer: c) They both converge to the golden ratio.

18. What is the product of the first four Lucas numbers?

- a) 36
- b) 72
- c) 144
- d) 288

Hint: Multiply the first four numbers in the Lucas sequence.

Answer: c) 144

19. Which of the following is a property unique to the Fibonacci sequence and not the Lucas sequence?

- a) Exponential growth
- b) Alternation between odd and even
- c) Convergence to a constant
- d) Relationship to the golden ratio

Hint: Look for distinctive characteristics of the Fibonacci sequence.

Answer: b) Alternation between odd and even

20. What is the 12th Lucas number?

- a) 322
- b) 528
- c) 856
- d) 1380

Hint: Count through the Lucas sequence to find the 12th number.

Answer: b) 528

21. Which number is common in both the Fibonacci and Lucas sequences?

- a) 13
- b) 21
- c) 34
- d) 55

Hint: Check which number appears in both sequences.

Answer: c) 34

22. What is the sum of the first seven Fibonacci numbers?

- a) 20
- b) 33
- c) 55
- d) 89



Fermat Numbers

1. What is the formula for Fermat numbers?

- A) $2^n - 1$
- B) $2^{(2^n)} - 1$
- C) $n^2 + 1$
- D) $2^{(n+1)}$

Hint: Think about the form of Fermat numbers as given by the formula.

Answer: B

2. How many Fermat numbers are there below 100?

- A) 3
- B) 5
- C) 7
- D) 10

Hint: Calculate the Fermat numbers for small values of n.

Answer: B

3. Which mathematician first introduced Fermat numbers?

- A) Pierre-Simon Laplace
- B) Carl Friedrich Gauss
- C) Pierre de Fermat

D) Leonard Euler

Hint: Fermat numbers are named after this mathematician.

Answer: C

4. What is the relationship between Fermat numbers and Fermat primes?

- A) They are the same
- B) Fermat primes are a subset of Fermat numbers
- C) Fermat numbers are a subset of Fermat primes
- D) No relationship

Hint: Fermat primes are a specific type of Fermat numbers.

Answer: B

5. Which of the following is a Fermat prime?

- A) 2
- B) 3
- C) 5
- D) 17

Hint: Fermat primes are prime numbers of the form $2^{2^n} + 1$.

Answer: D

6. What is the smallest known value of n for which $2^{2^n} + 1$ is not prime?

- A) 1
- B) 2
- C) 3
- D) 4

Hint: Check the primality of Fermat numbers for small n .

Answer: C

7. What is the general form of Fermat numbers?

- A) $3^n - 1$
- B) $2^{(2^n)} - 1$
- C) $n^2 + 1$
- D) $2^{(n+1)}$

Hint: Consider the exponentiation in the formula.

Answer: B

8. Which branch of mathematics is closely related to Fermat numbers?

- A) Geometry
- B) Number Theory
- C) Algebra
- D) Calculus

Hint: Fermat numbers are studied within this mathematical branch.

Answer: B

9. What is the second Fermat number?

- A) 3
- B) 5
- C) 7
- D) 17

Hint: Substitute $n=1$ into the Fermat number formula.

Answer: A

10. Which famous mathematician posed Fermat's Last Theorem?

- A) Pierre-Simon Laplace
- B) Carl Friedrich Gauss
- C) Pierre de Fermat
- D) Andrew Wiles

Hint: Fermat's Last Theorem is named after this mathematician.

Answer: C

11. What is the only known Fermat number that is a perfect square?

- A) 3
- B) 5
- C) 7
- D) None

Hint: Consider the values of n for which Fermat numbers are perfect squares.

Answer: D

12. Which of the following is a necessary condition for a Fermat number to be prime?

- A) Odd exponent
- B) Even exponent
- C) Prime exponent
- D) Composite exponent

Hint: Examine the exponents of known Fermat primes.

Answer: B

13. What is the largest known Fermat prime?

- A) 3
- B) 5
- C) 17
- D) 257

Hint: Look for the largest known prime among Fermat numbers.

Answer: D

14. What is the sum of the first three Fermat numbers?

- A) 7
- B) 15
- C) 31
- D) 63

Hint: Calculate the first three Fermat numbers and add them.

Answer: C

15. Which of the following statements about Fermat numbers is true?

- A) All Fermat numbers are prime.
- B) Fermat numbers are always even.
- C) There are infinitely many Fermat numbers.
- D) Fermat numbers are only defined for odd exponents.

Hint: Consider the primality and parity of Fermat numbers.

Answer: C

16. What is the digit sum of the 5th Fermat number $(2^{(2^4)} + 1)$?

- A) 7
- B) 8

- C) 9
- D) 10

Hint: Calculate the 5th Fermat number and sum its digits.

Answer: B

17. Which of the following is a factor of all Fermat numbers?

- A) 2
- B) 3
- C) 5
- D) 7

Hint: Check the divisibility of Fermat numbers by small primes.

Answer: A

18. What is the smallest known exponent for which $2^{(2^n)} + 1$ is composite?

- A) 1
- B) 2
- C) 3
- D) 4

Hint: Investigate the composite nature of Fermat numbers for small n .

Answer: C

19. Which mathematician proved Fermat's Last Theorem in 1994?

- A) Pierre-Simon Laplace
- B) Carl Friedrich Gauss
- C) Pierre de Fermat
- D) Andrew Wiles

Hint: This mathematician solved Fermat's Last Theorem after centuries.

Answer: D

20. What is the pattern of the last digit in Fermat numbers for increasing n ?

- A) 1, 3, 7, 9
- B) 2, 4, 8, 6
- C) 3, 6, 9, 2
- D) 5, 0, 5, 0

Hint: Observe the last digits of Fermat numbers for different values of n .

Answer: B

GREATEST COMMON DIVISOR

1. What does GCD stand for in number theory?

- A) Greatest Counting Divisor
- B) Generalized Common Denominator
- C) Greatest Common Divisor
- D) Grand Central Division

Hint: Think about what the G and C represent.

Answer: C

2. If the GCD of two numbers is 1, what can be said about those numbers?

- A) They are both even
- B) They are both prime
- C) They are both odd
- D) They are both composite

Hint: What kind of numbers have a GCD of 1 with each other?

Answer: B

3. What is the GCD of 24 and 36?

- A) 6
- B) 8
- C) 12
- D) 24

Hint: List the factors of both numbers and find the greatest common one.

Answer: A

4. If the GCD of two numbers is not 1, what do those numbers have in common?

- A) They are both even
- B) They are both prime
- C) They are both odd
- D) They are both composite

Hint: GCD reflects a shared factor.

Answer: D

5. What is the GCD of any two consecutive numbers?

- A) 1
- B) 2
- C) 3
- D) The GCD is undefined for consecutive numbers

Hint: Consider the relationship between consecutive numbers.

Answer: A

THE EUCLIDEAN ALGORITHM

1. What is the GCD of 15 and 20?

- A) 3
- B) 5
- C) 10
- D) 15

Hint: Consider the common factors of both numbers.

Answer: B

2. Which of the following is always true for the GCD of two numbers?

- A) GCD is always prime
- B) GCD is always composite
- C) GCD is always a factor of both numbers
- D) GCD is always larger than both numbers

Hint: Think about the definition of the GCD.

Answer: C

3. What is the GCD of 36 and 48?

- A) 6
- B) 12

C) 18

D) 24

Hint: Find the common factors and choose the largest one.

Answer: B

4. If the GCD of two numbers is 1, what can be said about the numbers?

A) They are both even

B) They are both prime

C) They are both odd

D) They are both composite

Hint: What does a GCD of 1 imply about the common factors?

Answer: B

5. What is the GCD of any number and 1?

A) 0

B) 1

C) The number itself

D) Undefined

Hint: Consider the definition of GCD.

Answer: B

6. If the GCD of two numbers is 5 and their product is 150, what are the numbers?

- A) 5 and 25
- B) 10 and 15
- C) 3 and 50
- D) 2 and 75

Hint: Use the relation between GCD and the product of numbers.

Answer: C

7. What is the GCD of any two consecutive numbers?

- A) 1
- B) 2
- C) The smaller number
- D) The larger number

Hint: Think about the factors of consecutive numbers.

Answer: A

8. What is the GCD of any number and 0?

- A) 0
- B) 1
- C) The number itself

D) Undefined

Hint: Consider the definition of GCD.

Answer: C

9. If the GCD of two numbers is 7 and their LCM is 56, what are the numbers?

- A) 7 and 8
- B) 7 and 49
- C) 14 and 28
- D) 14 and 7

Hint: Use the relationship between GCD and LCM.

Answer: C

10. What is the GCD of any number and its square?

- A) 1
- B) The number itself
- C) The square root of the number
- D) 2

Hint: Consider the factors of the given number and its square.

Answer: B

LEAST COMMON MULTIPLE

1. What does LCM stand for in number theory?

- A) Least Common Measure
- B) Lowest Common Multiple
- C) Littlest Common Multiplier
- D) Last Common Divisor

Hint: LCM involves finding a shared multiple.

Answer: B

2. Which of the following is the LCM of 6 and 8?

- A) 12
- B) 14
- C) 16
- D) 24

Hint: Find the common multiple that is the smallest.

Answer: D

3. What is the LCM of 3, 5, and 7?

- A) 105
- B) 120

C) 140

D) 150

Hint: Find the product of the prime factors raised to their highest powers.

Answer: A

4. If the LCM of two numbers is 36, and one number is 9, what is the other number?

A) 2

B) 3

C) 4

D) 6

Hint: Use the formula $\text{LCM}(a, b) = (a * b) / \text{GCD}(a, b)$.

Answer: C

5. Which of the following is true about the LCM of any set of numbers?

A) Always greater than or equal to the largest number

B) Always smaller than the smallest number

C) Always a prime number

D) Can be any real number

Hint: Consider the nature of LCM and how it relates to the given set of numbers.

Answer: A

CONGRUENCES

1. What does it mean for two integers a and b to be congruent modulo m ?

- A) a is divisible by b
- B) $a - b$ is divisible by m
- C) $a + b$ is divisible by m
- D) a and b are prime numbers

Hint: Think about the remainder when a is divided by m .

Answer: B

2. In congruence notation, what does $(a \equiv b \pmod{m})$ mean?

- A) a is not congruent to b
- B) a is congruent to b modulo m
- C) a is divisible by m
- D) a is a prime number

Hint: Focus on the relationship between a and b modulo m .

Answer: B

Sure, here are 10 multiple-choice questions on congruences in number theory, along with hints and answers:

1. What does it mean for integers a , b , and m to satisfy the congruence $a \equiv b \pmod{m}$?

- A) a is divisible by b
- B) a and b leave the same remainder when divided by m
- C) a and b are both prime
- D) a is a multiple of m

Hint: Think about the concept of congruence and remainders.

Answer: B

2. Which notation represents the congruence relation?

- A) $a \equiv b \pmod{m}$
- B) $a = b \pmod{m}$
- C) $a \sim b \pmod{m}$
- D) $a : b \pmod{m}$

Hint: Look for the standard notation used in number theory.

Answer: A

3. What is the solution to the congruence $7x \equiv 3 \pmod{5}$?

- A) $x \equiv 2 \pmod{5}$
- B) $x \equiv 4 \pmod{5}$
- C) $x \equiv 1 \pmod{5}$
- D) $x \equiv 3 \pmod{5}$

Hint: Use modular arithmetic to find the value of x .

Answer: B

4. When is the solution to the congruence $ax \equiv b \pmod{m}$ guaranteed to exist?

- A) Always
- B) Only if a and b are prime
- C) Only if a and m are coprime
- D) Never

Hint: Consider the relationship between a and m .

Answer: C

5. What is the Chinese Remainder Theorem used for in number theory?

- A) Factoring large numbers
- B) Solving systems of simultaneous congruences
- C) Calculating prime numbers
- D) Finding modular inverses

Hint: Think about its application in solving congruences.

Answer: B

6. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, what can you conclude about $a + c \equiv b + d \pmod{m}$?

- A) It is true
- B) It is false
- C) It depends on the values of a , b , c , and d
- D) Only true if $a = c$ and $b = d$

Hint: Consider the properties of congruences with addition.

Answer: A

*7. What is Euler's totient function, $\phi(n)$, used to calculate?

- A) The sum of divisors of n
- B) The number of integers coprime to n
- C) The greatest common divisor of n
- D) The smallest prime factor of n

Hint: Think about the function's relationship to coprime numbers.

Answer: B

8. Which of the following is true about Fermat's Little Theorem?

- A) It is used to find prime numbers
- B) It is a generalization of the Chinese Remainder Theorem
- C) It is a special case of Euler's totient function

D) It states that if p is prime, then $a^{(p-1)} \equiv 1 \pmod{p}$ for any a not divisible by p

Hint: Consider the theorem's statement and its conditions.

Answer: D

9. What is the modular inverse of $3 \pmod{11}$?

- A) 1
- B) 4
- C) 7
- D) 10

Hint: Find the number x such that $3 * x \equiv 1 \pmod{11}$.

Answer: C

10. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, what can you conclude about $ac \equiv bd \pmod{m}$?

- A) It is true
- B) It is false
- C) It depends on the values of a , b , c , and d
- D) Only true if $a = c$ and $b = d$

Hint: Consider the properties of congruences with multiplication.

Answer: A

THE POLLARD RHO FACTORING METHOD

1. What type of algorithm is the Pollard Rho algorithm in number theory?

- A) Primality testing
- B) Factoring
- C) Encryption
- D) Sorting

Hint: The Pollard Rho algorithm is specifically designed for a certain mathematical operation.

Answer: B

2. What is the main advantage of the Pollard Rho algorithm over other factoring methods?

- A) It always guarantees a quick factorization
- B) It works efficiently for large composite numbers
- C) It only works for prime numbers
- D) It is a deterministic algorithm

Hint: Consider the scalability of the algorithm and its efficiency for large numbers.

Answer: B

3. Which mathematical concept does the Pollard Rho algorithm exploit to factorize numbers?

- A) Prime factorization
- B) Quadratic residues
- C) Continued fractions

D) Elliptic curves

Hint: The algorithm uses a concept related to numbers' behavior under a certain operation.

Answer: C

4. In the Pollard Rho algorithm, what is the role of the "tortoise" and "hare" metaphor?

- A) It describes the speed of the algorithm
- B) It represents two different factorization methods
- C) It visualizes the algorithm's random walk behavior
- D) It symbolizes the algorithm's complexity

Hint: Think about how the algorithm iteratively progresses.

Answer: C

5. What is the time complexity of the Pollard Rho algorithm for factoring large numbers?

- A) $O(\log n)$
- B) $O(n)$
- C) $O(n \log n)$
- D) $O(\sqrt{n})$

Hint: Consider how the efficiency of the algorithm scales with the size of the number to be factored.

Answer: D

1. What is the Pollard Rho method primarily used for in number theory?

- A) Prime factorization
- B) Calculating square roots
- C) Finding prime numbers
- D) Solving Diophantine equations

Hint: The method is specifically designed for one of these tasks in number theory.

Answer: A

2. Who developed the Pollard Rho factorization method?

- A) John Pollard
- B) Peter Shor
- C) Carl Pomerance
- D) Richard Brent

Hint: The method is named after its creator.

Answer: A

3. What type of number does the Pollard Rho method work particularly well for factoring?

- A) Odd composite numbers
- B) Even composite numbers
- C) Prime numbers
- D) Perfect squares

Hint: The method has a specific efficiency for a certain category of numbers.

Answer: A

4. How does the Pollard Rho method handle the task of factoring large numbers?

- A) Utilizes elliptic curves
- B) Employs random walks in a function
- C) Applies quadratic residues
- D) Solves modular arithmetic equations

Hint: Think about the approach involving random walks.

Answer: B

5. In what time complexity class does the Pollard Rho method typically fall?

- A) $O(\log n)$
- B) $O(n)$
- C) $O(n^2)$
- D) $O(\sqrt{n})$

Hint: Consider the efficiency of the algorithm in terms of the input size.

Answer: D

THREE CLASSICAL MILESTONES

1. Who is often considered the "Father of Number Theory"?

- A) Euclid
- B) Pythagoras
- C) Archimedes
- D) Diophantus

Hint: He is known for his work "Elements."

Answer: A

2. Which ancient mathematician is credited with the Pythagorean theorem?

- A) Euclid
- B) Pythagoras
- C) Archimedes
- D) Diophantus

Hint: His name is synonymous with the theorem.

Answer: B

3. What is the fundamental theorem of arithmetic?

- A) Every integer is divisible by 2

- B) Every integer is a prime number
- C) Every positive integer can be uniquely expressed as a product of primes
- D) Every positive integer is a perfect square

Hint: It involves the factorization of integers.

Answer: C

4. Who introduced the concept of congruence in number theory?

- A) Euler
- B) Fermat
- C) Gauss
- D) Diophantus

Hint: His work laid the foundation for modular arithmetic.

Answer: C

5. Which theorem states that there are infinitely many prime numbers?

- A) Euclid's Lemma
- B) Goldbach's Conjecture
- C) Fermat's Last Theorem
- D) Euclid's Prime Number Theorem

Hint: It's an ancient theorem by Euclid.

Answer: A

WILLSON'S THEOREM

1. What does Wilson's Theorem state?

- A) Every integer is a prime number.
- B) Every prime number is odd.
- C) Every integer n greater than 1 is a divisor of $(n - 1)! + 1$.
- D) Every even integer is a perfect square.

Hint: Think about the relationship between factorials and divisibility.

Answer: C

2. In Wilson's Theorem, what is the condition for the integer 'p'?

- A) p is even.
- B) p is a prime number.
- C) p is an odd number.
- D) p is a composite number.

Hint: Wilson's Theorem is specifically applicable to a certain type of integer.

Answer: B

3. What is the value of $(4! + 1)$ in the context of Wilson's Theorem?

- A) 23
- B) 24

C) 120

D) 121

Hint: Apply Wilson's Theorem to find the value.

Answer: D

4. Can Wilson's Theorem be used to determine whether a number is prime or not?

A) Yes, for all numbers.

B) Yes, but only for odd numbers.

C) No, it only applies to specific cases.

D) No, it only works for even numbers.

Hint: Consider the specific condition of Wilson's Theorem.

Answer: C

5. Which mathematician is Wilson's Theorem named after?

A) John Wilson

B) Robert Wilson

C) John Wilson's student

D) John Wilson's mentor

Hint: The theorem is named after a specific mathematician.

Answer: A

1. What does Wilson's Theorem state?

- A) Every prime number is a factorial
- B) If p is prime, then $(p-1)! \equiv -1 \pmod{p}$
- C) If p is prime, then $p! \equiv -1 \pmod{p}$
- D) The sum of divisors of a prime number is -1

Hint: Wilson's Theorem provides a condition on factorials and primes.

Answer: B

2. What is the smallest prime for which Wilson's Theorem holds?

- A) 1
- B) 2
- C) 3
- D) 5

Hint: Consider the condition $(p-1)! \equiv -1 \pmod{p}$.

Answer: C

3. Which of the following is true according to Wilson's Theorem?

- A) $(4-1)! \equiv 3 \pmod{4}$

B) $(5-1)! \equiv 5 \pmod{5}$

C) $(6-1)! \equiv 5 \pmod{6}$

D) $(7-1)! \equiv 6 \pmod{7}$

Hint: Apply Wilson's Theorem to each case.

Answer: A

4. For which value of p does $(p-1)! \equiv -1 \pmod{p}$ fail?

A) $p = 1$

B) $p = 2$

C) $p = 3$

D) $p = 4$

Hint: Consider the condition in Wilson's Theorem.

Answer: D

5. How does Wilson's Theorem relate to primality testing?

A) It can be used to prove primality

B) It can be used to disprove primality

C) It is not related to primality

D) It only applies to even primes

Hint: Think about the implications of Wilson's Theorem on prime numbers.

Answer: A

FERMAT'S LITTLE THEOREM

1. What is Fermat's Last Theorem about?

- A) Prime numbers
- B) Perfect numbers
- C) Diophantine equations
- D) Pythagorean triples

Hint: It's a statement about a certain type of mathematical problem.

Answer: C

2. Who formulated Fermat's Last Theorem?

- A) Leonhard Euler
- B) Pierre-Simon Laplace
- C) Pierre de Fermat
- D) Andrew Wiles

Hint: The name is in the theorem's title.

Answer: C

3. Fermat's Last Theorem states that no three positive integers a , b , and c satisfy the equation $a^n + b^n = c^n$ for which value of n ?

- A) 1
- B) 2
- C) 3

D) 4

Hint: The theorem specifies a particular exponent.

Answer: D

4. What was the main challenge in proving Fermat's Last Theorem?

- A) Complex analysis
- B) Number theory
- C) Algebraic geometry
- D) Topology

Hint: The difficulty lies in a specific mathematical field.

Answer: C

5. Who successfully proved Fermat's Last Theorem, providing a complete and rigorous proof in 1994?

- A) Leonhard Euler
- B) Andrew Wiles
- C) Pierre de Fermat
- D) Carl Friedrich Gauss

Hint: The mathematician spent years on this famous problem.

Answer: B

6. Fermat's Last Theorem is a specific case of a more general problem known as what?

- A) Catalan's Conjecture
- B) Beal's Conjecture
- C) Goldbach's Conjecture
- D) Collatz Conjecture

Hint: It's another mathematical conjecture.

Answer: B

7. In Fermat's Last Theorem, what type of numbers are a , b , and c ?

- A) Rational numbers
- B) Imaginary numbers
- C) Real numbers
- D) Complex numbers

Hint: These numbers belong to a specific category.

Answer: A

8. Fermat's Last Theorem is often considered an unsolved problem in mathematics until when?

- A) 18th century
- B) 19th century
- C) 20th century
- D) 21st century

Hint: The solution came relatively recently.

Answer: C

9. Which branch of mathematics is closely related to Fermat's Last Theorem?

- A) Calculus
- B) Number theory
- C) Geometry
- D) Trigonometry

Hint: The problem involves integer solutions.

Answer: B

10. Fermat's Last Theorem is named "last" because it was the last theorem in Fermat's work that was unproven. True or False?

Hint: Consider Fermat's history with this theorem.

Answer: False

EULER'S THEOREM:

1. What does Euler's Theorem state?

- A) Every prime number has a unique factorization
- B) In a connected graph, the number of edges is equal to the number of vertices
- C) If a and n are coprime, then $a^{\phi(n)} \equiv 1 \pmod{n}$
- D) The sum of the angles in a triangle is 180 degrees

Hint: Euler's Theorem is related to modular arithmetic and coprime numbers.

Answer: C

2. In Euler's Theorem, what does $\phi(n)$ represent?

- A) Euler's constant
- B) Euler's totient function
- C) Euler's modulus
- D) Euler's exponent

Hint: $\phi(n)$ is a function related to the number of integers coprime to n .

Answer: B

3. What is the value of $\phi(12)$?

- A) 3
- B) 4

- C) 8
- D) 12

Hint: Count the numbers coprime to 12.

Answer: B

4. If $a \equiv 5 \pmod{11}$, what is $a^{\phi(11)} \pmod{11}$?

- A) 1
- B) 5
- C) 0
- D) 11

Hint: Apply Euler's Theorem with the given congruence.

Answer: A

5. Euler's Theorem is a generalization of:

- A) Fermat's Little Theorem
- B) Pythagorean Theorem
- C) Binomial Theorem
- D) Taylor's Theorem

Hint: Euler's Theorem shares a similarity with another famous theorem.

Answer: A

6. Which of the following numbers is always coprime to any positive integer n ?

- A) 0
- B) 1
- C) 2
- D) n

Hint: Coprime numbers have no common factors other than 1.

Answer: B

7. If $a \equiv 3 \pmod{7}$, what is $a^{\phi(7) - 1} \pmod{7}$?

- A) 1
- B) 3
- C) 2
- D) 0

Hint: Apply Euler's Theorem with the given congruence.

Answer: C

8. What is the smallest positive integer n for which $\phi(n) = 12$?

- A) 4
- B) 6

- C) 8
- D) 10

Hint: Find the number whose coprime count is 12.

Answer: D

9. Euler's Theorem is particularly useful in:

- A) Cryptography
- B) Calculus
- C) Geometry
- D) Probability

Hint: Think about applications that involve modular arithmetic.

Answer: A

10. If $a \equiv 2 \pmod{5}$, what is $a^{\phi(5) + 1} \pmod{5}$?

- A) 2
- B) 1
- C) 0
- D) 4

Hint: Utilize Euler's Theorem with the given congruence.

Answer: B

FINITE CONTINUED FRATIONS

Question 1:

What is the value of the finite continued fraction $[2; 3, 4, 5]$?

- a) $2/15$
- b) $15/2$
- c) $3/15$
- d) $5/4$

Answer: b) $15/2$

Hint: To find the value of a finite continued fraction, work from the innermost fraction outward. In this case, start with the last term (5) and move towards the first term (2).

1. Question:

$$x = 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4}}}$$

What is the value of x ?

- a) $\frac{25}{16}$
- b) $\frac{16}{25}$
- c) $\frac{5}{4}$
- d) $\frac{4}{5}$

Hint: Start by evaluating the innermost fraction.

Answer: a) $\frac{25}{16}$

2. Question:

If $[a; b, c, d]$ represents the finite continued fraction $a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}$, what is the value of $[2; 3, 4, 5]$?

- a) $\frac{120}{77}$
- b) $\frac{77}{120}$
- c) $\frac{5}{4}$
- d) $\frac{4}{5}$

Hint: Evaluate the expression step by step.

Answer: b) $\left(\frac{77}{120} \right)$

3. Question:

If $x = 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$, what is the value of x ?

- a) $\frac{3125}{1456}$
- b) $\frac{1456}{3125}$
- c) $\frac{5}{4}$
- d) $\frac{4}{5}$

Hint: Work from the innermost fraction outward.

Answer: a) $\left(\frac{3125}{1456} \right)$

4. Question:

Evaluate the finite continued fraction $\left([1; 1, 1, 1, 1, 1] \right)$.

- a) $\frac{1}{2}$
- b) $\frac{3}{4}$
- c) $\frac{1}{4}$
- d) $\frac{3}{2}$

Hint: Sum the fractions iteratively.

Answer: c) $\frac{1}{4}$

5. Question:

If $[a; b, c] = 2$, and $[a; b, c, d] = 3$, what is the value of d ?

- a) $\frac{5}{4}$
- b) $\frac{4}{5}$
- c) $\frac{5}{2}$
- d) $\frac{2}{5}$

Hint: Use the information about the values of $[a; b, c]$ and $[a; b, c, d]$.

Answer: b) $\frac{4}{5}$

6. Question:

Evaluate the finite continued fraction $[2; 1, 3, 1, 2]$.

- a) $\frac{11}{9}$
- b) $\frac{9}{11}$
- c) $\frac{12}{5}$
- d) $\frac{4}{5}$

Hint: Work through the fractions step by step.

Answer: a) $\frac{11}{9}$

7. Question:

If $x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$, what is the value of x ?

- a) $\frac{3}{2}$
- b) $\frac{5}{3}$
- c) $\frac{7}{4}$
- d) $\frac{9}{5}$

Hint: Simplify the expression.

Answer: c) $\frac{7}{4}$

8. Question:

If $\frac{1}{a+b} = \frac{7}{3}$, what is the value of $\frac{1}{a+b+c}$ if $\frac{1}{c} = 2$?

- a) $\frac{27}{11}$
- b) $\frac{11}{27}$
- c) $\frac{5}{4}$
- d) $\frac{4}{5}$

Hint: Use the information about the values of $\frac{1}{a+b}$ and $\frac{1}{c}$.

Answer: a) $\frac{27}{11}$

9. Question:

Evaluate the finite continued fraction $[1; 2, 1, 2, 1]$.

- a) $\frac{5}{3}$

b) $\frac{3}{5}$

c) $\frac{7}{4}$

d) $\frac{4}{7}$

Hint: Work through the fractions step by step.

Answer: c) $\left(\frac{7}{4}\right)$

10. Question:

If $x = 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5 + \frac{1}{6}}}}}$, what is the value of x ?

a) $\frac{40320}{18199}$

b) $\frac{18199}{40320}$

c) $\frac{7}{4}$

d) $\frac{4}{7}$

Hint: Evaluate the expression step by step.

Answer: a) $40320/18199$

Question 1:

Express the golden ratio, ϕ , as an infinite continued fraction.

Hint: Remember that the continued fraction for the golden ratio is often written as

$$\phi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}$$

2 **Answer:** $\phi = [1; 1, 1, 1, \dots]$

Question: What is the continued fraction expansion of the golden ratio, ϕ ?

- a) $\sqrt{2}$
- b) $\frac{1}{\sqrt{2}}$
- c) $\frac{1+\sqrt{5}}{2}$
- d) $\frac{\sqrt{5}-1}{2}$

Answer: c

Hint: Recall the definition of the golden ratio and its relationship to its continued fraction expansion.

3. The continued fraction representation of $5/2$ is:

- a) $2; 2, 2, 2, \dots; 2, 2, 2, \dots$
- b) $2; 1, 2, 3, \dots; 2, 1, 2, 3, \dots$
- c) $2; 1, 4, 1, 4, \dots; 2, 1, 4, 1, 4, \dots$
- d) $2; 2, 3, 4, \dots; 2, 3, 4, \dots$

Answer: c

Hint: Express $5/2$ as a sum of integers and then find its continued fraction representation.

4.

Question: What is the value of the infinite continued fraction $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$?

- a) 1
- b) 2
- c) $\sqrt{2}$
- d) $\frac{1+\sqrt{5}}{2}$

Answer: d

Hint: Consider the relationship between the continued fraction and the golden ratio.

5.

Question: Which of the following is a finite continued fraction?

- a) $2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}$
- b) $3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$
- c) $\sqrt{2} + \frac{1}{\sqrt{2} + \frac{1}{\sqrt{2} + \frac{1}{\sqrt{2}}}}$
- d) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$

Answer: a

Hint: A finite continued fraction has a limited number of terms.

6. Question: The continued fraction representation of $\sqrt{3}$ is:

- a) $1;1,1,1,\dots;1,1,1,\dots$
- b) $2;1,2,1,2,\dots;2,1,2,1,2,\dots$
- c) $1;2,1,2,\dots;1,2,1,2,\dots$
- d) $1;1,2,1,2,\dots;1,1,2,1,2,\dots$

Answer: b

Hint: Express $\sqrt{3}$ as a sum of integers and find its continued fraction representation.

1. What is the value of the infinite continued fraction $[1; 1, 1, 1, 1, \dots]$?

- a) 0
- b) 1
- c) $\pi/2$
- d) e

Hint: Consider the pattern of the continued fraction and try to express it in a familiar mathematical constant.

Answer: c) $\pi/2$

2. Evaluate the infinite continued fraction $[2; 2, 2, 2, 2, \dots]$.

- a) 1
- b) 2
- c) 4
- d) $\sqrt{2}$

Hint: Think about how the repetition of the same value affects the continued fraction.

Answer: b) 2-

3. What is the value of $[3; 1, 2, 3, 4, \dots]$?

- a) e
- b) $\sqrt{2}$
- c) $\pi/2$
- d) 2

Hint: Observe the relationship between the terms and try to simplify the expression.

Answer: a) e

4. Evaluate the infinite continued fraction $[0; 1, 1, 1, 1, \dots]$.

- a) 0
- b) 1
- c) 2
- d) e

Hint: Consider the meaning of the initial 0 and its impact on the continued fraction.

Answer: a) 0

5. Find the value of $[1; 2, 3, 4, 5, \dots]$.

- a) $\sqrt{2}$
- b) e
- c) π
- d) Golden Ratio (ϕ)

Hint: Look for a connection with a well-known mathematical constant.

Answer: d) Golden Ratio (ϕ)

6. Evaluate the infinite continued fraction $[1; 1, 1/2, 1/3, 1/4, \dots]$.

- a) e
- b) π
- c) $\ln(2)$
- d) 2

Hint: Consider the reciprocals of the terms and see if there's a familiar series.

Answer: c) $\ln(2)$

7. What is the value of $[2; 1, 2, 1, 2, \dots]$?

- a) 2
- b) 3
- c) 4
- d) 5

Hint: Notice the pattern of alternating terms.

Answer: b) 3

8. Evaluate the infinite continued fraction $[1; 2, 1, 2, 1, \dots]$.

- a) $\sqrt{2}$
- b) e
- c) π
- d) $\ln(2)$

Hint: Look for a repeating pattern and simplify accordingly.

Answer: a) $\sqrt{2}$

9. Find the value of $[1; 1, 1/3, 1/5, 1/7, \dots]$.

- a) $\pi/2$
- b) e
- c) $\ln(2)$
- d) Golden Ratio (ϕ)

Hint: Consider the reciprocals of the odd integers.

Answer: a) $\pi/2$

10. What is the value of $[2; 1, 1, 1, 1, \dots]$?

- a) 1
- b) 2

c) e

d) π

Hint: Observe the pattern of repeating 1s.

Answer: b) 2

1

What is the value of the infinite continued fraction $[1; 1, 1, 1, \dots]$?

a) $\frac{1}{2}$

b) $\frac{\sqrt{5}-1}{2}$

c) $\sqrt{2}$

d) e

Answer: b) $\frac{\sqrt{5}-1}{2}$

Hint: Consider the equation $x = 1 + \frac{1}{x}$ and solve for x .

Hint: Consider the equation $x = 1 + \frac{1}{x}$ and solve for x .

2. Question:

What is the value of the infinite continued fraction $[2; 2, 2, 2, \dots]$?

a) $2/3$

b) $3/2$

c) 2

d) $\sqrt{2}$

Answer: c) 2

Hint: Evaluate the infinite continued fraction by simplifying the expression repeatedly.

3. Question:

What is the continued fraction representation of the square root of 3?

- a) $\sqrt{[1; 1, 1, 1, \dots]}$
- b) $\sqrt{[1; 2, 1, 2, 1, \dots]}$
- c) $\sqrt{[1; 1, 2, 1, 2, \dots]}$
- d) $\sqrt{[1; 1, 1, 2, 2, \dots]}$

Answer: c) $\sqrt{[1; 1, 2, 1, 2, \dots]}$

Hint: Use the fact that $\sqrt{3} = 1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}$.

4. Question:

What is the value of the infinite continued fraction $\sqrt{[3; 3, 3, 3, \dots]}$?

- a) $3/2$
- b) $2/3$
- c) 3
- d) $\sqrt{3}$

Answer: a) $3/2$

Hint: Evaluate the continued fraction by simplifying the expression repeatedly.

5. Question:

What is the continued fraction representation of the golden ratio ϕ ?

- a) $\{ [1; 2, 1, 2, 1, \dots] \}$
- b) $\{ [1; 1, 1, 1, \dots] \}$
- c) $\{ [1; 1, 2, 1, 2, \dots] \}$
- d) $\{ [2; 1, 1, 1, \dots] \}$

Answer: a) $\{ [1; 2, 1, 2, 1, \dots] \}$

Hint: Use the fact that $\phi = 1 + \frac{1}{1 + \frac{1}{1 + \dots}}$.

6. Question:

What is the value of the infinite continued fraction $\{ [4; 4, 4, 4, \dots] \}$?

- a) $\{ 4 \}$
- b) $4/3$
- c) $3/4$
- d) $\sqrt{4}$

Answer: a) 4

Hint: Evaluate the continued fraction by simplifying the expression repeatedly.

7. Question:

What is the value of the infinite continued fraction $\{ [1; 2, 2, 2, \dots] \}$?

- a) $\sqrt{2}$
- b) $\sqrt{3}$
- c) $\sqrt{5}$
- d) $\sqrt{7}$

Answer: a) $\sqrt{2}$

$$x = 1 + \frac{2}{x}$$

Hint: Consider the equation and solve for x .

8. Question:

What is the value of the infinite continued fraction $[2; 1, 1, 1, \dots]$?

- a) $\frac{\sqrt{5}-1}{2}$
- b) $\frac{1}{\sqrt{2}}$
- c) $\frac{1}{\sqrt{3}}$
- d) $\frac{1}{\sqrt{5}}$

Answer: b) $\frac{1}{\sqrt{2}}$

Hint: Evaluate the continued fraction by simplifying the expression repeatedly.

9. Question:

What is the value of the infinite continued fraction $[3; 1, 3, 1, 3, \dots]$?

- a) $\sqrt{3}$
- b) $\sqrt{7}$
- c) $\sqrt{10}$
- d) $\sqrt{13}$

Answer: a) $\sqrt{3}$

Hint: Use the fact that $\sqrt{3} = 1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \dots}}}}$.

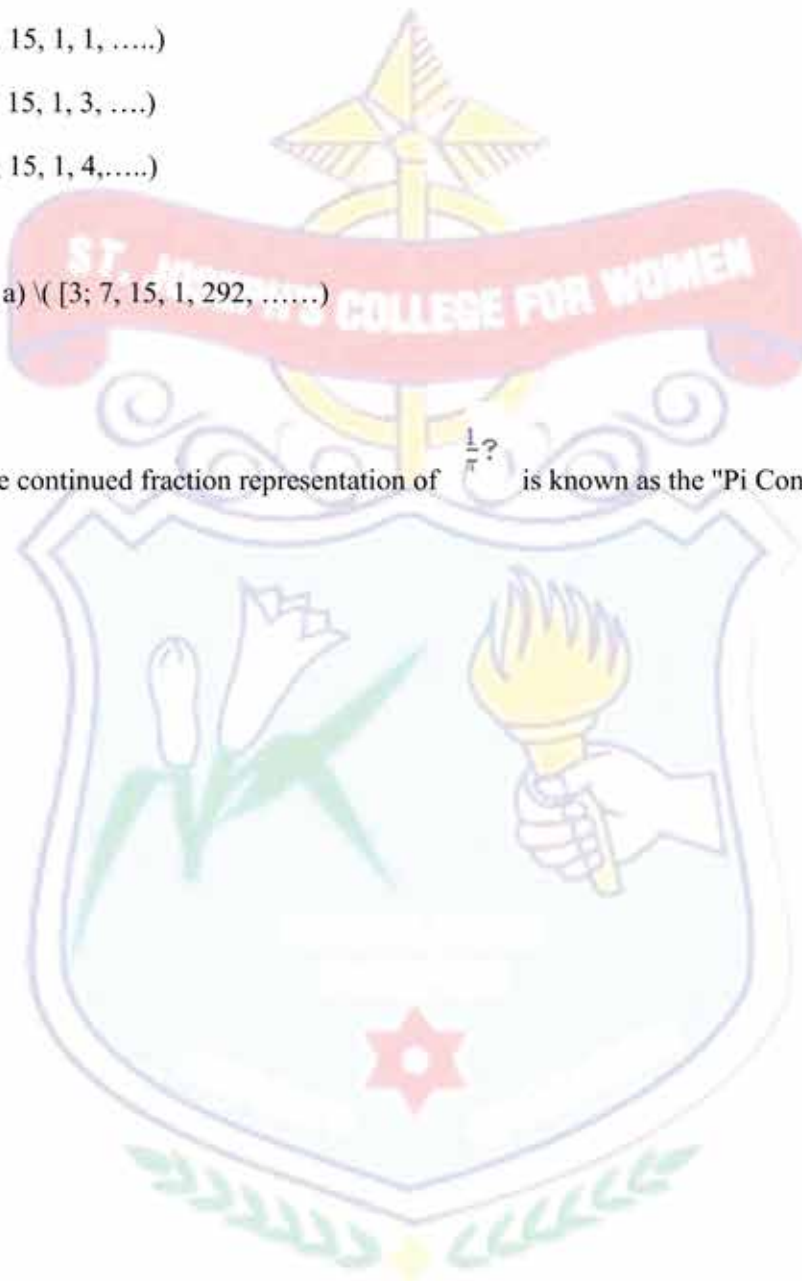
10. Question:

What is the continued fraction representation of $\frac{1}{\pi}$?

- a) $\{ [3; 7, 15, 1, 292, \dots] \}$
- b) $\{ [3; 7, 15, 1, 1, \dots] \}$
- c) $\{ [3; 7, 15, 1, 3, \dots] \}$
- d) $\{ [3; 7, 15, 1, 4, \dots] \}$

Answer: a) $\{ [3; 7, 15, 1, 292, \dots] \}$

Hint: The continued fraction representation of $\frac{1}{\pi}$ is known as the "Pi Continued Fraction."



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